

THE PEP STANDARD
COMPUTABLE GENERAL EQUILIBRIUM MODEL
SINGLE-COUNTRY, STATIC VERSION
PEP-1-1

VERSION 2.0 ¹

MAY 2012

EQUATIONS, SETS, VARIABLES AND PARAMETERS

Bernard Decaluwé ²

André Lemelin ³

Véronique Robichaud ⁴

Hélène Maisonnave ⁵



-
- ¹ Version 2.0 refers to version 2.0 of the GAMS code. This document succeeds and replaces the January 2010 edition with November 2010 minor corrections.
- ² Département d'économique, Université Laval, Québec
- ³ Centre INRS-UCS, Université du Québec, Montréal
- ⁴ CIRPÉE et PEP, Université Laval, Québec
- ⁵ Financial and Fiscal Commission, South Africa, and PEP, Université Laval, Québec



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc-nd/3.0/>

APPENDIX A: EQUATIONS, SETS, VARIABLES AND PARAMETERS

A1. Equations

A1.1 PRODUCTION

1. $VA_j = v_j XST_j$
2. $CI_j = io_j XST_j$
3. $VA_j = B_j^{VA} \left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + (1 - \beta_j^{VA}) KDC_j^{-\rho_j^{VA}} \right]^{\frac{1}{\rho_j^{VA}}}$
4. $LDC_j = \left[\frac{\beta_j^{VA} RC_j}{1 - \beta_j^{VA} WC_j} \right]^{\sigma_j^{VA}} KDC_j$
5. $LDC_j = B_j^{LD} \left[\sum_l \beta_{l,j}^{LD} LD_{l,j}^{-\rho_j^{LD}} \right]^{\frac{1}{\rho_j^{LD}}}$
6. $LD_{l,j} = \left[\frac{\beta_{l,j}^{LD} WC_j}{WTI_{l,j}} \right]^{\sigma_j^{LD}} (B_j^{LD})^{\sigma_j^{LD} - 1} LDC_j$
7. $KDC_j = B_j^{KD} \left[\sum_k \beta_{k,j}^{KD} KD_{k,j}^{-\rho_j^{KD}} \right]^{\frac{1}{\rho_j^{KD}}}$
8. $KD_{k,j} = \left[\frac{\beta_{k,j}^{KD} RC_j}{RTI_{k,j}} \right]^{\sigma_j^{KD}} (B_j^{KD})^{\sigma_j^{KD} - 1} KDC_j$
9. $DI_{i,j} = aij_{i,j} CI_j$

A1.2 INCOME AND SAVINGS

A1.2.1 Households

10. $YH_h = YHL_h + YHK_h + YHTR_h$
11. $YHL_h = \sum_l \lambda_{h,l}^{WL} \left(W_l \sum_j LD_{l,j} \right)$
12. $YHK_h = \sum_k \lambda_{h,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$
13. $YHTR_h = \sum_{ag} TR_{h,ag}$
14. $YDH_h = YH_h - TDH_h - TR_{gvt,h}$

$$15. CTH_h = YDH_h - SH_h - \sum_{agng} TR_{agng,h}$$

$$16. SH_h = PIXCON^n sh0_h + sh1_h YDH_h$$

A1.2.2 Firms

$$17. YF_f = YFK_f + YFTR_f$$

$$18. YFK_f = \sum_k \lambda_{f,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$$

$$19. YFTR_f = \sum_{ag} TR_{f,ag}$$

$$20. YDF_f = YF_f - TDF_f$$

$$21. SF_f = YDF_f - \sum_{ag} TR_{ag,f}$$

A1.2.3 Government

$$22. YG = YGK + TDHT + TDFT + TPROD + TPRCTS + YGTR$$

$$23. YGK = \sum_k \lambda_{gvt,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$$

$$24. TDHT = \sum_h TDH_h$$

$$25. TDFT = \sum_f TDF_f$$

$$26. TPROD = TIWT + TIKT + TIPT$$

$$27. TIWT = \sum_{l,j} TIW_{l,j}$$

$$28. TIKT = \sum_{k,j} TIK_{k,j}$$

$$29. TIPT = \sum_j TIP_j$$

$$30. TPRCTS = TICT + TIMT + TIXT$$

$$31. TICT = \sum_i TIC_i$$

$$32. TIMT = \sum_i TIM_i$$

$$33. TIXT = \sum_i TIX_i$$

$$34. YGTR = \sum_{agng} TR_{gvt,agng}$$

$$35. TDH_h = PIXCON^n ttdh0_h + ttdh1_h YH_h$$

$$36. TDF_f = PIXCON^n tdf0_f + tdf1_f YFK_f$$

$$37. TIW_{l,j} = ttiw_{l,j} W_l LD_{l,j}$$

$$38. TIK_{k,j} = ttik_{k,j} R_{k,j} KD_{k,j}$$

$$39. TIP_j = tip_j PP_j XST_j$$

$$40. TIC_i = ttic_i \left[\left(PL_i + \sum_{ij} PC_{ij} tmg_{ij,i} \right) DD_i + \left((1 + tim_i) PWM_i e + \sum_{ij} PC_{ij} tmg_{ij,i} \right) IM_i \right]$$

$$41. TIM_i = tim_i PWM_i e IM_i$$

$$42. TIX_i = tix_i \left(PE_i + \sum_{ij} PC_{ij} tmg_{ij,i}^X \right) EXD_i$$

$$43. SG = YG - \sum_{agn} TR_{agn,gvt} - G$$

A1.2.4 Rest of the world

$$44. YROW = e \sum_i PWM_i IM_i + \sum_k \lambda_{row,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right) + \sum_{agd} TR_{row,agd}$$

$$45. SROW = YROW - \sum_i PE_i^{FOB} EXD_i - \sum_{agd} TR_{agd,row}$$

$$46. SROW = -CAB$$

A1.2.5 Transfers

$$47. TR_{agn,h} = \lambda_{agn,h}^{TR} YDH_h$$

$$48. TR_{gvt,h} = PIXCON^\eta tr0_h + tr1_h YH_h$$

$$49. TR_{ag,f} = \lambda_{ag,f}^{TR} YDF_f$$

$$50. TR_{agn,gvt} = PIXCON^\eta TR_{agn,gvt}^0$$

$$51. TR_{agd,row} = PIXCON^\eta TR_{agd,row}^0$$

A1.3 DEMAND

$$52. PC_i C_{i,h} = PC_i C_{i,h}^{MIN} + \gamma_{i,h}^{LES} \left(CTH_h - \sum_{ij} PC_{ij} C_{ij,h}^{MIN} \right)$$

$$53. GFCF = IT - \sum_i PC_i VSTK_i$$

$$54. PC_i INV_i = \gamma_i^{INV} GFCF$$

$$55. PC_i CG_i = \gamma_i^{GVT} G$$

$$56. DIT_i = \sum_j DI_{i,j}$$

$$57. MRGN_i = \sum_{ij} tmg_{i,ij} DD_{ij} + \sum_{ij} tmg_{i,ij} IM_{ij} + \sum_{ij} tmg_{i,ij}^X EXD_{ij}$$

A1.4 PRODUCER SUPPLIES OF PRODUCTS AND INTERNATIONAL TRADE

$$58. XST_j = B_j^{XT} \left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}} \right]^{\frac{1}{\rho_j^{XT}}}$$

$$59. XS_{j,i} = \frac{XST_j}{(B_j^{XT})^{1+\sigma_j^{XT}}} \left[\frac{P_{j,i}}{\beta_{j,i}^{XT} PT_j} \right]^{\sigma_j^{XT}}$$

$$60. XS_{j,i} = B_{j,i}^X \left[\beta_{j,i}^X EX_{j,i}^{\rho_{j,i}^X} + (1 - \beta_{j,i}^X) DS_{j,i}^{\rho_{j,i}^X} \right]^{\frac{1}{\rho_{j,i}^X}}$$

$$61. EX_{j,i} = \left[\frac{1 - \beta_{j,i}^X \frac{PE_i}{PL_i}}{\beta_{j,i}^X \frac{PE_i}{PL_i}} \right]^{\sigma_{j,i}^X} DS_{j,i}$$

$$62. EXD_i = EXD_i^O \left(\frac{e PWX_i}{PE_i^{FOB}} \right)^{\sigma_i^{XD}}$$

$$63. Q_i = B_i^M \left[\beta_i^M IM_i^{-\rho_i^M} + (1 - \beta_i^M) DD_i^{-\rho_i^M} \right]^{\frac{-1}{\rho_i^M}}$$

$$64. IM_i = \left[\frac{\beta_i^M \frac{PD_i}{PM_i}}{1 - \beta_i^M \frac{PD_i}{PM_i}} \right]^{\sigma_i^M} DD_i$$

A1.5 PRICES

A1.5.1 Production

$$65. PP_j = \frac{PVA_j VA_j + PCI_j CI_j}{XST_j}$$

$$66. PT_j = (1 + ttip_j) PP_j$$

$$67. PCI_j = \frac{\sum_i PC_i DI_{i,j}}{CI_j}$$

$$68. PVA_j = \frac{WC_j LDC_j + RC_j KDC_j}{VA_j}$$

$$69. WC_j = \frac{\sum_l WTI_{l,j} LD_{l,j}}{LDC_j} \text{ (redundant, given equations 5 and 6; see Appendix E2)}$$

$$70. WTI_{l,j} = W_l (1 + ttiw_{l,j})$$

$$71. RC_j = \frac{\sum_k RTI_{k,j} KD_{k,j}}{KDC_j} \text{ (redundant, given equations 7 and 8; see Appendix E3)}$$

72. $RTI_{k,j} = R_{k,j} (1 + ttik_{k,j})$
 73. $R_{k,j} = RK_k$, if capital is mobile

A1.5.2 International trade

74. $PT_j = \frac{\sum_i P_{j,i} XS_{j,i}}{XST_j}$ (redundant, given equations 58 and 59; see Appendix E4)
75. $P_{j,i} = \frac{PE_i EX_{j,i} + PL_i DS_{j,i}}{XS_{j,i}}$
76. $PE_i^{FOB} = \left(PE_i + \sum_{ij} PC_{ij} tmrg_{ij,i}^X \right) (1 + tix_i)$
77. $PD_i = (1 + ttic_i) \left(PL_i + \sum_{ij} PC_{ij} tmrg_{ij,i} \right)$
78. $PM_i = (1 + ttic_i) \left((1 + ttim_i) e PWM_i + \sum_{ij} PC_{ij} tmrg_{ij,i} \right)$
79. $PC_i = \frac{PM_i IM_i + PD_i DD_i}{Q_i}$

A1.5.3 Price indexes

80. $PIXGDP = \sqrt{\frac{\sum_j PVA_j VAO_j}{\sum_j PVAO_j VAO_j} \frac{\sum_j PVA_j VA_j}{\sum_j PVAO_j VA_j}}$
81. $PIXCON = \frac{\sum_i PC_i \sum_h C_{i,h}^0}{\sum_{ij} PC_{ij}^0 \sum_h C_{ij,h}^0}$
82. $PIXINV = \prod_i \left(\frac{PC_i}{PC_i^0} \right)^{\gamma_i^{INV}}$
83. $PIXGVT = \prod_i \left(\frac{PC_i}{PC_i^0} \right)^{\gamma_i^{GVT}}$

A1.6 EQUILIBRIUM

84. $Q_i = \sum_h C_{i,h} + CG_i + INV_i + VSTK_i + DIT_i + MRGN_i$
85. $\sum_j LD_{l,j} = LS_l$
86. $\sum_j KD_{k,j} = KS_k$

$$87. IT = \sum_h SH_h + \sum_f SF_f + SG + SROW$$

$$88. \sum_j DS_{j,i} = DD_i$$

$$89. \sum_j EX_{j,i} = EXD_i$$

A1.7 GROSS DOMESTIC PRODUCT

$$90. GDP^{BP} = \sum_j PVA_j VA_j + TIPT$$

$$91. GDP^{MP} = GDP^{BP} + TPRCTS$$

$$92. GDP^{IB} = \sum_{l,j} W_l LD_{l,j} + \sum_{k,j} R_{k,j} KD_{k,j} + TPROD + TPRCTS$$

$$93. GDP^{FD} = \sum_i PC_i \left[\sum_h C_{i,h} + CG_i + INV_i + VSTK_i \right] + \sum_i PE_i^{FOB} EXD_i - e \sum_i PWM_i IM_i$$

A2. Sets

A2.1 INDUSTRIES AND COMMODITIES

All industries: $j, jj \in J = \{J_1, \dots, J_j, \dots\}$

All commodities: $i, ij \in I = \{I_1, \dots, I_i, \dots\}$

A2.2 PRODUCTION FACTORS

Labor categories: $l \in L = \{L_1, \dots, L_l, \dots\}$

Capital categories: $k \in K = \{K_1, \dots, K_k, \dots\}$

A2.3 AGENTS

All agents: $ag, agj \in AG = H \cup F \cup \{GVT, ROW\} = \{H_1, \dots, H_h, \dots, F_1, \dots, F_f, \dots, GVT, ROW\}$

Household categories: $h, hj \in H \subset AG = \{H_1, \dots, H_h, \dots\}$

Firm categories: $f, fj \in F \subset AG = \{F_1, \dots, F_f, \dots\}$

Non governmental agent:

$agng \in AGNG \subset AG = H \cup F \cup \{ROW\} = \{H_1, \dots, H_h, \dots, F_1, \dots, F_f, \dots, ROW\}$

Domestic agents: $agd \in AGD \subset AG = H \cup F \cup \{GVT\} = \{H_1, \dots, H_h, \dots, F_1, \dots, F_f, \dots, GVT\}$

A3. Variables

NOTE: In what follows, the word “taxes” should be understood as “taxes, minus subsidies”.

A3.1 VOLUME VARIABLES

$C_{i,h}$	Consumption of commodity i by type h households
$C_{i,h}^{MIN}$	Minimum consumption of commodity i by type h households
CG_i	Public consumption of commodity i
CI_j	Total intermediate consumption of industry j
DD_i	Domestic demand for commodity i produced locally
$DI_{i,j}$	Intermediate consumption of commodity i by industry j
DIT_i	Total intermediate demand for commodity i
$DS_{j,i}$	Supply of commodity i by sector j to the domestic market
$EX_{j,i}$	Quantity of product i exported by sector j
EXD_i	World demand for exports of product i
IM_i	Quantity of product i imported
INV_i	Final demand of commodity i for investment purposes
$KD_{k,j}$	Demand for type k capital by industry j
KDC_j	Industry j demand for composite capital
KS_k	Supply of type k capital
$LD_{l,j}$	Demand for type l labor by industry j
LDC_j	Industry j demand for composite labor
LS_l	Supply of type l labor
$MRGN_i$	Demand for commodity i as a trade or transport margin
Q_i	Quantity demanded of composite commodity i
VA_j	Value added of industry j
$VSTK_i$	Inventory change of commodity i
$XS_{j,i}$	Industry j production of commodity i
XST_j	Total aggregate output of industry j

A3.2 PRICE VARIABLES

e	Exchange rate ⁶ ; price of foreign currency in terms of local currency
-----	---

⁶ The default choice of numeraire in PEP-1-1 is the exchange rate e . This is implemented by fixing the value of e as exogenous. But the choice of numeraire in a CGE model is arbitrary

$P_{j,i}$:	Basic price of industry j 's production of commodity i
PC_i :	Purchaser price of composite commodity i (including all taxes and margins)
PCI_j :	Intermediate consumption price index of industry j
PD_i :	Price of local product i sold on the domestic market (including all taxes and margins)
PE_i :	Price received for exported commodity i (excluding export taxes)
PE_i^{FOB} :	FOB price of exported commodity i (in local currency)
$PIXCON$:	Consumer price index
$PIXGDP$:	GDP deflator
$PIXGVT$:	Public expenditures price index
$PIXINV$:	Investment price index
PL_i :	Price of local product i (excluding all taxes on products)
PM_i :	Price of imported product i (including all taxes and tariffs)
PP_j :	Industry j unit cost, including taxes directly related to the use of capital and labor, but excluding other taxes on production
PT_j :	Basic price of industry j 's output
PVA_j :	Price of industry j value added (including taxes on production directly related to the use of capital and labour)
PWM_i :	World price of imported product i (expressed in foreign currency)
PWX_i :	World price of exported product i (expressed in foreign currency)
$R_{k,j}$:	Rental rate of type k capital in industry j
RC_j :	Rental rate of industry j composite capital
RK_k :	Rental rate of type k capital (if capital is mobile)
$RTI_{k,j}$:	Rental rate paid by industry j for type k capital, including capital taxes
W_l :	Wage rate of type l labor
WC_j :	Wage rate of industry j composite labor
$WTI_{l,j}$:	Wage rate paid by industry j for type l labor, including payroll taxes

A3.3 NOMINAL (VALUE) VARIABLES

CAB :	Current account balance
CTH_h :	Consumption budget of type h households

(although the interpretation of results can be more or less easy, depending on which numeraire is selected).

G :	Current government expenditures on goods and services
GDP^{BP} :	GDP at basic prices
GDP^{FD} :	GDP at purchasers' prices from the perspective of final demand
GDP^{IB} :	GDP at market prices (income-based)
GDP^{MP} :	GDP at market prices
$GFCF$:	Gross fixed capital formation
IT :	Total investment expenditures
SF_f :	Savings of type f businesses
SG :	Government savings
SH_h :	Savings of type h households
$SROW$:	Rest-of-the-world savings
TDF_f :	Income taxes of type f businesses
$TDFT$:	Total government revenue from business income taxes
TDH_h :	Income taxes of type h households
$TDHT$:	Total government revenue from household income taxes
TIC_i :	Government revenue from indirect taxes on product i
$TICT$:	Total government receipts of indirect taxes on commodities
$TIK_{k,j}$:	Government revenue from taxes on type k capital used by industry j
$TIKT$:	Total government revenue from from taxes on capital
TIM_i :	Government revenue from import duties on product i
$TIMT$:	Total government revenue from import duties
TIP_j :	Government revenue from taxes on industry j production (excluding taxes directly related to the use of capital and labor)
$TIPT$:	Total government revenue from production taxes (excluding taxes directly related to the use of capital and labor)
$TIW_{l,j}$:	Government revenue from payroll taxes on type l labor in industry j
$TIWT$:	Total government revenue from payroll taxes
TIX_i :	Government revenue from export taxes on product i
$TIXT$:	Total government revenue from export taxes
$TPRCTS$:	Total government revenue from taxes on products and imports
$TPRODN$:	Total government revenue from other taxes on production ⁷

⁷ That is, taxes on production other than taxes on products and taxes and duties on imports (see Appendix B1).

$TR_{ag,agj}$	Transfers from agent agj to agent ag
YDF_f	Disposable income of type f businesses
YDH_h	Disposable income of type h households
YF_f	Total income of type f businesses
YFK_f	Capital income of type f businesses
$YFTR_f$	Transfer income of type f businesses
YG	Total government income
YGK	Government capital income
$YGTR$	Government transfer income
YH_h	Total income of type h households
YHK_h	Capital income of type h households
YHL_h	Labor income of type h households
$YHTR_h$	Transfer income of type h households
$YROW$	Rest-of-the-world income

A3.4 RATES AND INTERCEPTS

$sh0_h$	Intercept (type h household savings)
$sh1_h$	Slope (type h household savings)
$tr0_h$	Intercept (transfers by type h households to government)
$tr1_h$	Marginal rate of transfers by type h households to government
$ttdf0_f$	Intercept (income taxes of type f businesses)
$ttdf1_f$	Marginal income tax rate of type f businesses
$ttdh0_h$	Intercept (income taxes of type h households)
$ttdh1_h$	Marginal income tax rate of type h households
$ttic_i$	Tax rate on commodity i
$ttik_{k,j}$	Tax rate on type k capital used in industry j
$ttim_i$	Rate of taxes and duties on imports of commodity i
$ttip_j$	Tax rate on the production of industry j
$ttiw_{l,j}$	Tax rate on type l worker compensation in industry j
$ttix_i$	Export tax rate on exported commodity i

A4. Parameters

$aij_{i,j}$	Input-output coefficient
B_j^{KD}	Scale parameter (CES – composite capital)
B_j^{LD}	Scale parameter (CES – composite labor)
B_i^M	Scale parameter (CES – composite commodity)
B_j^{VA}	Scale parameter (CES – value added)
$B_{j,i}^X$	Scale parameter (CET – exports and local sales)
B_j^{XT}	Scale parameter (CET – total output)
$\beta_{k,j}^{KD}$	Share parameter (CES – composite capital)
$\beta_{l,j}^{LD}$	Share parameter (CES – composite labor)
β_i^M	Share parameter (CES – composite commodity)
β_j^{VA}	Share parameter (CES – value added)
$\beta_{j,i}^X$	Share parameter (CET – exports and local sales)
$\beta_{j,i}^{XT}$	Share parameter (CET – total output)
η	Price elasticity of indexed transfers and parameters
γ_i^{GVT}	Share of commodity i in total current public expenditures on goods and services
γ_i^{INV}	Share of commodity i in total investment expenditures
$\gamma_{i,h}^{LES}$	Marginal share of commodity i in type h household consumption budget
io_j	Coefficient (Leontief – intermediate consumption)
$\lambda_{ag,k}^{RK}$	Share of type k capital income received by agent ag
$\lambda_{ag,agj}^{TR}$	Share parameter (transfer functions)
$\lambda_{h,l}^{WL}$	Share of type l labor income received by type h households
ρ_j^{KD}	Elasticity parameter (CES – composite capital); $-1 < \rho_j^{KD} < \infty$
ρ_j^{LD}	Elasticity parameter (CES – composite labor); $-1 < \rho_j^{LD} < \infty$
ρ_i^M	Elasticity parameter (CES – composite commodity); $-1 < \rho_i^M < \infty$
ρ_j^{VA}	Elasticity parameter (CES – value added); $-1 < \rho_j^{VA} < \infty$
$\rho_{j,i}^X$	Elasticity parameter (CET – exports and local sales); $1 < \rho_{j,i}^X < \infty$
ρ_j^{XT}	Elasticity parameter (CET – total output); $1 < \rho_j^{XT} < \infty$
σ_j^{KD}	Elasticity of substitution (CES – composite capital); $0 < \sigma_j^{KD} < \infty$
σ_j^{LD}	Elasticity of substitution (CES – composite labor); $0 < \sigma_j^{LD} < \infty$

- σ_i^M : Elasticity of substitution (CES – composite commodity); $0 < \sigma_i^M < \infty$
 σ_j^{VA} : Elasticity of transformation (CES – value added) ; $0 < \sigma_j^{VA} < \infty$
 $\sigma_{j,i}^X$: Elasticity of transformation (CET – exports and local sales) ; $0 < \sigma_{j,i}^X < \infty$
 σ_i^{XD} : Price elasticity of the world demand for exports of product i
 σ_j^{XT} : Elasticity of transformation (CET – total output) ; $0 < \sigma_j^{XT} < \infty$
 $tmrg_{i,ij}$: Rate of margin i applied to commodity ij
 $tmrg_{i,ij}^X$: Rate of margin i applied to exported commodity ij
 v_j : Coefficient (Leontief – value added)