Functional Forms Commonly Used in CGE models: Exercises

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Please send your answer (*doc, or pdf format, scanned version of written notes are accepted*) to training@agrodep.org indicating in the email object: [MIRAGRODEP TRAINING].

1 Demand functions

1.1 Marshallian and Hicksian demand functions

Let R be the agent's income; there are n commodities (i=1,2,…n) which this agent buys in quantity q_i, the price of good i being p_i. Derive the Marshallian, then the Hicksian demand functions for the following utility type:

a. Cobb-Douglas: $U = \prod_{i=1}^{n} q_i^{\alpha_i}$ with $\alpha_i > 0, \forall i$

b. CES: $U = \left(\sum_{i=1}^{n} \alpha_i q_i^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}}$

c. LES-CES: $U = \left(\sum_{i=1}^{n} \alpha_i^{\frac{1}{\sigma}} (q_i - qmin_i)^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}}$

1.2 Compensated price elasticities

Derive the compensated own price and cross price elasticities for the following demand systems:

a. Cobb-Douglas
b. CES
c. LES-CES

1.3 Translog demand system

The quadratic logarithmic indirect utility function is given by:

$$\ln \Psi\,(p,R) = \alpha_0 + \sum_k \alpha_k \ln\left(\frac{p_k}{R}\right) + 0.5 \sum_k \sum_j \gamma_{kj} \ln\left(\frac{p_k}{R}\right) \ln\left(\frac{p_j}{R}\right)$$

a. Using Roy's Identity, find the corresponding share equations. This is called the Translog demand system.
b. Derive the uncompensated price elasticities
c. Derive the income elasticities

2 Production functions

Y is the quantity produced, P the market price of the output, x_i and w_i being the quantity used and price of input i. The technology is $Y = F(x_1; x_2; \dots; x_n)$

2.1 Marginal Rate of Technical Substitution

The Marginal Rate of Technical Substitution (MRTS) is the amount by which the quantity of one input has to be reduced when one extra unit of another input is used, so that output remains constant.

$$MRTS(x_i, x_j) = \frac{\frac{\partial F}{\partial x_i}}{\frac{\partial F}{\partial x_j}}$$

Derive the MRTS for the following production function:

a. Cobb Douglas: $Y = F(x_1; x_2; \dots; x_n) = A \prod_i {x_i}^{\alpha_i}$

b. CES: $Y = F(x_1; x_2; \dots; x_n) = A \left(\sum_{i=1}^{n} \alpha_i {x_i}^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}}$ avec A>0, α_i>0 et $\sum_i \alpha_i = 1$.

c. CET: $Y = F(x_1; x_2; \dots; x_n) = A \left(\sum_{i=1}^{n} \alpha_i {x_i}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}}$ avec A>0, α_i>0 et $\sum_i \alpha_i = 1$.

d. Leontief $Y = F(x_1; x_2; \dots; x_n) = min\left\{\frac{x_i}{\alpha_i}\right\}$

2.2 Elasticity of substitution

The elasticity of substitution between production factors is derived like this

$$\sigma_{ij} = \frac{\partial \ln\left(\frac{x_j}{x_i}\right)}{\partial \ln\left(MRTS(x_i, x_j)\right)}$$

Derive the elasticity of substitution for the following production technologies:

a. Cobb Douglas
b. CES
c. CET
d. Leontief