Functional Forms Commonly Used in CGE models: Solutions

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Please send your questions (*doc, or pdf format, scanned version of written notes are accepted***) to training@agrodep.org indicating in the email object: [MIRAGRODEP TRAINING].**

1 Demand functions

1.1 Marshallian and Hicksian demand functions

The Marshallian demand functions are:

a. Cobb-Douglas

$$q_i = \frac{R\alpha_i}{p_i}$$

b. CES

$$q_i = \frac{{\alpha_i}^{\sigma} R}{{p_i}^{\sigma} \sum_j {\alpha_j}^{\sigma} {p_j}^{1-\sigma}}$$

c. LES-CES

$$q_i = qmin_i + \frac{\alpha_i\left(R - \sum_j p_j qmin_j\right)}{{p_i}^{\sigma} \sum_j \alpha_j {p_j}^{1-\sigma}}$$

The Hicksian demand functions are:

a. Cobb-Douglas

$$q_i = u\frac{\alpha_i}{p_i}\prod_j \left(\frac{p_j}{\alpha_j}\right)^{\alpha_j}$$

b. CES

$$q_i = u\frac{\alpha_i}{{p_i}^{\sigma}}\left(\sum_j \frac{\alpha_j}{{p_j}^{\sigma-1}}\right)^{\frac{\sigma}{1-\sigma}}$$

c. LES-CES

$$q_i = qmin_i + u\frac{\alpha_i}{{p_i}^{\sigma}}\left(\sum_j \frac{\alpha_j}{{p_j}^{\sigma-1}}\right)^{\frac{\sigma}{1-\sigma}}$$

1.2 Compensated price elasticities

a. Cobb-Douglas

$$\varepsilon^{H}{}_{i} = \alpha_i - 1$$

$$\varepsilon^{H}{}_{ij} = \alpha_j$$

b. CES

$$\varepsilon^{H}{}_{i} = \sigma\left[\frac{\alpha_i p_i{}^{1-\sigma}}{(\sum_k \alpha_k p_k{}^{1-\sigma})} - 1\right] = \sigma\left(\frac{p_i q_i}{R} - 1\right)$$

$$\varepsilon^{H}{}_{ijr} = \sigma\frac{q_j p_j}{R}$$

c. LES-CES

$$\varepsilon^{H}{}_{i} = \sigma\frac{\alpha_i p_i{}^{1-\sigma}}{q_i \sum \alpha_k p_k{}^{1-\sigma}}\left[\left((q_i - qmin_i) - \frac{(R - \sum p_k qmin_k)}{p_i}\right)\right]$$

$$\varepsilon^{H}{}_{ij} = \sigma\frac{\alpha_i}{p_i{}^{\sigma}\sum \alpha_k p_k{}^{1-\sigma}}\frac{p_j\left(q_j - qmin_{jr}\right)}{q_i} = \sigma\frac{(q_i - qmin_i)}{q_i}\frac{p_j(q_j - qmin_j)}{(R - \sum p_k qmin_k)}$$

1.3 Translog demand system

$$\ln \mathrm{V}\,(p, R) = \alpha_0 + \sum_k \alpha_k \ln\left(\frac{p_k}{R}\right) + 0.5\sum_k\sum_j \gamma_{kj}\ln\left(\frac{p_k}{R}\right)\ln\left(\frac{p_j}{R}\right)$$

a. Roy's identity:

$$q_i = -\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial R}}$$

Logarithmic version of Roy's identity:

$$q_i = -\frac{\frac{\partial V}{\partial p_i}\frac{1}{V}}{\frac{\partial V}{\partial R}\frac{1}{V}} = -\frac{\frac{\partial \ln V}{\partial p_i}}{\frac{\partial \ln V}{\partial R}}$$

$$\frac{\partial \ln V}{\partial p_i} = \frac{\alpha_i}{p_i} + \frac{1}{p_i}\sum_k \gamma_{ki}\ln\left(\frac{p_k}{R}\right)$$

$$\frac{\partial \ln V}{\partial R} = -\frac{1}{R}\sum_k \left(\alpha_k + \sum_j \gamma_{kj} \ln\left(\frac{p_j}{R}\right)\right)$$

$$\Rightarrow q_i = \frac{\frac{\alpha_i}{p_i} + \frac{1}{p_i}\sum_k \gamma_{ki} \ln\left(\frac{p_k}{R}\right)}{\frac{1}{R}\sum_k \left(\alpha_k + \sum_j \gamma_{kj} \ln\left(\frac{p_j}{R}\right)\right)}$$

$$\frac{p_i q_i}{R} = \frac{\alpha_i + \sum_k \gamma_{ki} \ln\left(\frac{p_k}{R}\right)}{\sum_k \left(\alpha_k + \sum_j \gamma_{kj} \ln\left(\frac{p_j}{R}\right)\right)}$$

b.

$$\varepsilon_i = \frac{\gamma_{ii}/w_i - \sum_k \gamma_{ik}}{-1 + \sum_l \sum_k \gamma_{lk} \ln\left(\frac{p_k}{R}\right)} - 1$$

$$\varepsilon_{ij} = \frac{\gamma_{ij}/w_i - \sum_k \gamma_{ik}}{-1 + \sum_l \sum_k \gamma_{lk} \ln\left(\frac{p_k}{R}\right)}$$

c.

$$\eta_i = 1 - \frac{\sum_k \gamma_{ik}/w_i - \sum_l \sum_k \gamma_{lk}}{-1 + \sum_l \sum_k \gamma_{lk} \ln\left(\frac{p_k}{R}\right)}$$

2 Production functions

2.1 Marginal Rate of Technical Substitution

a.

$$MRTS(x_i, x_j) = \frac{\alpha_i x_j}{\alpha_j x_i}$$

b.

$$MRTS(x_i, x_j) = \frac{\alpha_i}{\alpha_j}\left(\frac{x_i}{x_j}\right)^{-\frac{1}{\sigma}}$$

c.

$$MRTS(x_i, x_j) = \frac{\alpha_i}{\alpha_j}\left(\frac{x_i}{x_j}\right)^{\frac{1}{\sigma}}$$

d.

$MRTS(x_i, x_j) = \infty$ if $x_i < x_j$ and $MRTS(x_i, x_j) = 0$ if $x_i > x_j$ (not defined for $x_i = x_j$)

2.2 Elasticity of substitution

a.

$$MRTS(x_i, x_j) = \frac{\alpha_i x_j}{\alpha_j x_i}$$

$$\Rightarrow \frac{x_j}{x_i} = \frac{\alpha_j MRTS(x_i, x_j)}{\alpha_i}$$

$$\Rightarrow \ln\left(\frac{x_j}{x_i}\right) = \ln\left(\frac{\alpha_j}{\alpha_i}\right) + \ln\left(MRTS(x_i, x_j)\right)$$

$$\Rightarrow \sigma_{ij} = \frac{\partial \ln\left(\frac{x_j}{x_i}\right)}{\partial \ln\left(MRTS(x_i, x_j)\right)} = 1$$

b.

$$\sigma_{ij} = \frac{\partial \ln\left(\frac{x_j}{x_i}\right)}{\partial \ln\left(MRTS(x_i, x_j)\right)} = \sigma$$

c.

$$\sigma_{ij} = -\sigma$$

d.

$$\sigma_{ij} = 0$$