Applied Panel Data Analysis – Lecture 8

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- We will discuss endogeneity in the unobserved effects model
- Traditional endogeneity where some of the covariates are correlated with the idiosyncratic shocks in the model
- Endogeneity induced when some of the covariates are correlated with the unobserved effect while others are not

- In most applied economic settings endogeneity is considered the key hurdle for providing credible results
- Controlling for endogeneity allows one to progress from make statements about correlations to causation
- Endogeneity can arise from omitted variables, measurement error, sample selectivity, or self selection

- Consider our structural unobserved effects model

$$\begin{aligned} y_{it} &= Y_{1,it}'\gamma + x_{1,it}'\beta + c_i + \varepsilon_{it} \\ &= Z_{it}\delta + c_i + \varepsilon_{it} \end{aligned} \quad (1)$$

  where $Y_{1,it}$ are g_1 endogenous variables and $x_{1,it}$ are k_1 exogenous variables; $Z = [Y_1, X_1]$
- We also have $k_2 > g_1$ additional instrumental variables, $x_{2,it}$
- Let $x_{it} = [x_{1,it}, x_{2,it}]$ (so that $X = [X_1, X_2]$) denote the collection of all exogenous variables

- Endogeneity enters in the model when $E[\varepsilon_{it}|Z_{it}, c_i] \neq E[\varepsilon_{it}] = 0$
- Our instruments will satisfy the standard exogeneity condition $E[\varepsilon_{it}|X_{it}, c_i] = E[\varepsilon_{it}] = 0$
- Notice that our focus at the moment is with correlation between Y_1 and ε, the unobserved effect will be controlled through either the fixed or random effects framework

- Suppose we assume the fixed effects framework for our structural unobserved effects model
- Using the within transformation on (1) we have

$$Qy_{it} = QZ_{it}\delta + Q\varepsilon_{it} \tag{2}$$

- Using the instruments QX_{it}, two stage least squares estimation produces

$$\hat{\delta}_{W2SLS} = \left(\tilde{Z}'P_{\tilde{X}}\tilde{Z}\right)^{-1}\tilde{Z}'P_{\tilde{X}}\tilde{y} \tag{3}$$

  where $P_{\tilde{X}} = \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'$
- This is nothing more than 2SLS except we use as instruments $\tilde{X}$ instead of X

- The reason for the transformation is that the correlation between the fixed effects and the regressors needs to be removed prior to controlling for the correlation between Y_1 and X_2
- The variance-covariance matrix for $\hat{\delta}_{W2SLS}$ is

$$Var(\hat{\delta}_{W2SLS}) = \sigma_{\varepsilon}^2 \left(\tilde{Z}' P_{\tilde{X}} \tilde{Z} \right)^{-1} \tag{4}$$

- An alternative set of instruments is PX, which constitutes between estimation
- Using the between transformation on (1) we have

$$Py_{it} = PZ_{it}\delta + P\varepsilon_{it} \tag{5}$$

and two stage least squares estimation produces

$$\hat{\delta}_{B2SLS} = \left(\bar{Z}' P_{\bar{X}} \bar{Z}\right)^{-1} \bar{Z}' P_{\bar{X}} \bar{y} \tag{6}$$

where $P_{\bar{X}} = \bar{X}(\bar{X}'\bar{X})^{-1}\bar{X}'$

- The variance-covariance matrix for $\hat{\delta}_{B2SLS}$ is

$$Var(\hat{\delta}_{B2SLS}) = \sigma_1^2 \left(\bar{Z}' P_{\bar{X}} \bar{Z}\right)^{-1} \tag{7}$$

where $\sigma_1^2 = T\sigma_c^2 + \sigma_\varepsilon^2$

- If we focus on the one-way error component model, in the single equation setting our variance-covariance matrix is identical to the fully exogenous setting we discussed in Lecture 4
- Recall that GLS estimation of

$$\begin{pmatrix} Qy \\ Py \end{pmatrix} = \begin{pmatrix} QX \\ PX \end{pmatrix} \beta + \begin{pmatrix} Qu \\ Pu \end{pmatrix}$$

produced the one-way random effects estimator

- Now consider the following system of $2NT$ observations

$$\begin{pmatrix} X'Qy \\ X'Py \end{pmatrix} = \begin{pmatrix} X'QZ \\ X'PZ \end{pmatrix} \delta + \begin{pmatrix} X'Qu \\ X'Pu \end{pmatrix} \qquad (8)$$

- Given the validity of the instruments we have

$$E\left(\begin{array}{c} X'Qu \\ X'Pu \end{array}\right) = 0$$

and

$$Var\left(\begin{array}{c} X'Qu \\ X'Pu \end{array}\right) = \left[\begin{array}{cc} \sigma_{\varepsilon}^{2} X'QX & 0 \\ 0 & \sigma_{1}^{2} X'PX \end{array}\right] \quad (9)$$

- Thus, GLS estimation will produce an unbiased and consistent estimator of δ from (8)

- Baltagi (1981) derived the error component two-stage least squares estimator based on GLS estimation of (8):

$$\hat{\delta}_{EC2SLS} = \left[\frac{\tilde{Z}' P_{\tilde{X}} \tilde{Z}}{\sigma_\varepsilon^2} + \frac{\bar{Z}' P_{\bar{X}} \bar{Z}}{\sigma_1^2} \right]^{-1} \left[\frac{\tilde{Z}' P_{\tilde{X}} \tilde{y}}{\sigma_\varepsilon^2} + \frac{\bar{Z}' P_{\bar{X}} \bar{y}}{\sigma_1^2} \right] \tag{10}$$

- As in the fully exogenous case, the EC2SLS estimator can be succinctly written as a matrix weighted average of the B2SLS and FE2SLS estimators

$$\hat{\delta}_{EC2SLS} = W_1 \hat{\delta}_{FE2SLS} + W_2 \hat{\delta}_{B2SLS} \tag{11}$$

- Baltagi (1981) suggests consistent estimators for σ_ε^2 and σ_1^2 using the residual sum of squares from fixed effects two-stage least squares estimation and between two-stage least squares

$$\widehat{\sigma}_\varepsilon^2 = \frac{\hat{\varepsilon}'_{FE2SLS} Q \hat{\varepsilon}_{FE2SLS}}{N(T-1)} \tag{12}$$

$$\widehat{\sigma}_1^2 = \frac{\hat{\varepsilon}'_{B2SLS} P \hat{\varepsilon}_{B2SLS}}{N} \tag{13}$$

where $\hat{\varepsilon}_{FE2SLS} = y - Z\hat{\delta}_{FE2SLS}$ and $\hat{\varepsilon}_{B2SLS} = y - Z\hat{\delta}_{B2SLS}$

- An alternative two-stage least squares estimator for the one-way error component model is from Balestra and Varadharajan-Krisnakumar (1987)
- They suggest direct GLS estimation of (1) using $\Omega^{-1/2}$ with instruments $\Omega^{-1/2}X = \frac{\tilde{X}}{\sigma_\varepsilon} + \frac{\bar{X}}{\sigma_1}$
- Their generalized two-stage least squares estimator is

$$\hat{\delta}_{G2SLS} = \left(Z^{*\prime} P_{X^*} Z^*\right)^{-1} Z^{*\prime} P_{X^*} y^* \qquad (14)$$

- Note that Baltagi's (1981) EC2SLS estimator uses as instruments $[\tilde{X}, \bar{X}]$ while Balestra and Varadharajan-Krisnakumar's (1987) G2SLS estimator uses as instruments $\frac{\tilde{X}}{\sigma_\varepsilon} + \frac{\bar{X}}{\sigma_1}$
- How do these instrument sets differ?
- $[\tilde{X}, \bar{X}]$ spans a linear space of dimension $2(k_1 + k_2)$ while $\frac{\tilde{X}}{\sigma_\varepsilon} + \frac{\bar{X}}{\sigma_1}$ spans a linear space of dimension $k_1 + k_2$, i.e. the instrument set of Balestra and Varadharajan-Krisnakumar's (1987) is a subset of that of Baltagi (1981)
- Baltagi and Li (1992) show that in the single equation setting, these extra instruments do not yield reductions in the variance covariance matrix of $\hat{\delta}_{EC2SLS}$
- Moreover, $\hat{\delta}_{EC2SLS}$ and $\hat{\delta}_{G2SLS}$ have the same asymptotic variance-covariance matrix

- However, the use of $\hat{\hat{\delta}}_{EC2SLS}$ is still common because while the variance-covariance matrix is asymptotically the same as $\hat{\hat{\delta}}_{G2SLS}$ in the single equation setting, in the full system setting, Baltagi's approach yields gains in efficiency
- While not common, one should check estimates across the two instrument sets to see if there are any perceptible differences (there should not be except in perverse settings)

- Recall that the distinction between the random effects framework and the fixed effects framework was an all or nothing proposition
- Either all of the regressors were independent from the unobserved effect (random effects framework) or all regressors were allowed to be correlated with the unobserved effect (fixed effects framework)
- There was no middle ground for estimation between these two frameworks
- Hausman and Taylor (1981) proposed an estimation strategy that accomplished just this

- To begin, endogeneity will now exist in the unobserved effects model through correlation amongst a subset of the regressors and the unobserved effects
- An example is a wage regression where work experience and years of education are correlated with ability (lets assume it is time constant), which is part of the unobserved effect
- If we assume the fixed effects framework (which is feasible), but only these two variables are correlated with ability, then our structure is too strict

- The unobserved effects model of Hausman and Taylor (1981) is

$$y_{it} = x_{it}'\beta + z_i'\gamma + c_i + \varepsilon_{it} \tag{15}$$

- We further partition x_{it} and z_i into two pieces: $x_{it} = [x_{1,it}, x_{2,it}]$ and $z_i = [z_{1,i}, z_{2,i}]$
- $x_{1,it}$ is $k_1 \times 1$, $x_{2,it}$ is $k_2 \times 1$, $z_{1,i}$ is $g_1 \times 1$ and $z_{2,i}$ is $g_2 \times 1$
- We assume that x_1 and z_1 are exogenous with respect to both c and ε while x_2 and z_2 are exogenous with respect to ε but are endogenous with respect to c

- Notice that the within transformation would eliminate the endogeneity of x_2, but it also removes z_1 and z_2 from the model
- Hausman and Taylor's (1981) approach is to control for endogeneity without eliminating the time constant covariates from the model
- How do they do this?

- They suggest the standard random effects framework transformation $\Omega^{-1/2}$ to (15) and then application of two-stage least squares using as instruments

$$A = \left[\tilde{X}, PX_1, Z_1\right] \tag{16}$$

- Note that Z_1 instruments itself (since it is exogenous), while X_1 and X_2 are instrumented by $\tilde{X}$
- Z_2 is instrumented by PX_1; given the panel structure X_1 can be used in two different dimensions as an instrument

- As it stands the Hausman and Taylor (1981) procedure is infeasible given that the elements of Ω are unknown
- To construct a feasible estimator Hausman and Taylor propose the following approach
- First, estimate the model in (15) using the within transformation; this will naturally eliminate Z from the model so γ is not identified

- Second, average the residuals from within estimation of (15) across time

$$\hat{u}_{i.} = \bar{y}_{i.} - \bar{X}_{i.}\tilde{\beta} \tag{17}$$

- Third, perform two-stage least squares using instrument matrix $A = [X_1, Z_1]$ on the model

$$\hat{u}_{i.} = Z_i\gamma + \omega_i \tag{18}$$

- The estimator from this regression is

$$\hat{\gamma}_{2SLS} = (Z'P_AZ)^{-1}Z'P_A\hat{u} \tag{19}$$

- These steps provide consistent estimates of β and γ, which can be used to construct consistent estimates of σ_c^2 and σ_ε^2
- Consistent estimators of the variance components are

$$\hat{\sigma}_\varepsilon^2 = \frac{y' Q \left(I - P_{QX}\right) Q y}{N(T-1)} \tag{20}$$

and

$$\hat{\sigma}_1^2 = \frac{\left(y - X\tilde{\beta} - Z\hat{\gamma}_{2SLS}\right)' P \left(y - X\tilde{\beta} - Z\hat{\gamma}_{2SLS}\right)}{N} \tag{21}$$

- Using the variance component estimates the original unobserved effects model in (15) is transformed with $\hat{\Omega}^{-1/2}$ and two-stage least squares is performed using instrument matrix A from (16)
- If $k_1 < g_2$ then the model is under-identified, $\hat{\beta}_{HT} = \tilde{\beta}$ and $\hat{\gamma}_{HT}$ does not exist
- If $k_1 = g_2$ then the model is exactly-identified, $\hat{\beta}_{HT} = \tilde{\beta}$ and $\hat{\gamma}_{HT} = \hat{\gamma}_{2SLS}$
- If $k_1 > g_2$ then the model is over-identified, and $\hat{\beta}_{HT}$ is more efficient than $\tilde{\beta}$

- An over-identification test follows along the lines of the Hausman test of the random effects framework

$$\hat{m} = \left(\hat{\beta}_{HT} - \tilde{\beta}\right)' \left(var(\tilde{\beta}) - var(\hat{\beta}_{HT})\right)^{-1} \left(\hat{\beta}_{HT} - \tilde{\beta}\right) \quad (22)$$

- This statistic has limiting distribution χ^2_ℓ where $\ell = \min[k_1 - g_2, NT - (k_1 + k_2)]$
- This test allows one to discern if endogeneity in X_2 is severe
- Note that one only tests using β as the within transformation cannot identify γ

- Both Amemiya and MaCurdy (1986) and Breusch, Mizon and Schmidt (1989) proposed instrument sets that produce an estimator more efficient than the original Hausman and Taylor (1981) estimator
- These instrument sets are more likely to ensure identification of γ, however, they come at the expense of rapidly increasing the instrument set based on the time dimension of the panel
- Too many instruments can also be viewed negatively, even though there are efficiency gains to be had
- Further, there are additional exogeneity conditions that must be satisfied with these expanded instrument sets; whereas Hausman and Taylor (1981) only require that the time averaged X_1s are uncorrelated with c, the Amemiya and MaCurdy (1986) instrument set requires conditional strict exogeneity, a much stronger condition

- Discussed accounting for endogeneity in the unobserved effects model
- Estimation covered both the fixed and random effects framework
- Beyond endogeneity with the idiosyncratic error term, also discussed compromise between fixed and random effects framework that can allow for time constant variables
- Hausman-Taylor estimator allows endogeneity between covariates and unobserved effect; can identify time constant effects