Notes on QUAIDS Estimation

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1 QUAIDS Model

Based on [1] the QUAIDS expenditure share equation system is:

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[\frac{m}{a(\mathbf{p})} \right] + \frac{\lambda_i}{b(\mathbf{p})} \left\{ \ln \left[\frac{m}{a(\mathbf{p})} \right] \right\}^2 \tag{1}$$

Where

$$\ln a(\mathbf{p}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1} n \gamma_{ij} \ln p_i \ln p_j \tag{2}$$

$$b(\mathbf{p}) = \prod_{i=1}^{n} p_i^{\beta_i} \tag{3}$$

2 Adding-up, homogeneity and symmetry conditions

Adding up condition ($\sum w_i = 1$) implies:

$$\sum_{i=1}^{n} \alpha_i = 1 \qquad \sum_{i=1}^{n} \gamma_{ij} = 0 \qquad \sum_{i=1}^{n} \beta_i = 0 \qquad \sum_{i=1}^{n} \lambda_i = 0 \tag{4}$$

Since demand equations are homogenous of degree zero on m and $\mathbf{p}$ then the following condition must be satisfied:

$$\sum_{j=1} \gamma_{ij} = 0 \tag{5}$$

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For Slutsky symmetry $\left(\frac{\partial h_i(\mathbf{p},u)}{\partial p_j} = \frac{\partial h_j(\mathbf{p},u)}{\partial p_i}\right)$ we require:

$$\gamma_{ij} = \gamma_{ji} \tag{6}$$

3 Elasticities

To calculate QUAIDS model elasticities, differentiate equation (1) with respect to m and $\ln p_j$, respectively, to obtain

$$\mu_i \equiv \frac{\partial w_i}{\partial \ln m} = \beta_i + \frac{2\lambda_i}{b(\mathbf{p})}\left\{\ln\left[\frac{m}{a(\mathbf{p})}\right]\right\} \tag{7}$$

$$\mu_{ij} \equiv \frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \beta_i \frac{\partial \ln a(\mathbf{p})}{\partial \ln p_j} - \frac{2\lambda_i}{b(\mathbf{p})}\left\{\ln\left[\frac{m}{a(\mathbf{p})}\right]\right\}\frac{\partial \ln a(\mathbf{p})}{\partial \ln p_j} - \frac{\lambda_i}{b(\mathbf{p})^2}\frac{\partial b(\mathbf{p})}{\partial \ln p_j}\left\{\ln\left[\frac{m}{a(\mathbf{p})}\right]\right\}^2$$

where

$$\frac{\partial \ln a(\mathbf{p})}{\partial \ln p_j} = \alpha_j + \frac{1}{2}\sum_{k=1}^{n}\gamma_{kj}\ln p_k + \frac{1}{2}\sum_{k=1}^{n}\gamma_{jk}\ln p_k \tag{8}$$

using symmetry conditions:

$$\frac{\partial \ln a(\mathbf{p})}{\partial \ln p_j} = \alpha_j + \sum_{k=1}^{n}\gamma_{jk}\ln p_k \tag{9}$$

and

$$\frac{\partial b(\mathbf{p})}{\partial \ln p_j} = \frac{\partial \ln b(\mathbf{p})}{\partial \ln p_j} b(\mathbf{p}) = \beta_j b(\mathbf{p}) \tag{10}$$

then

$$\mu_{ij} \equiv \frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \mu_i\left(\alpha_j + \sum_{k=1}^{n}\gamma_{jk}\ln p_k\right) - \frac{\lambda_i\beta_j}{b(\mathbf{p})}\left\{\ln\left[\frac{m}{a(\mathbf{p})}\right]\right\}^2$$

Budget (income) elasticity:

$$
\begin{aligned}
\mu_i &= \frac{\partial \left(\frac{p_i q_i}{m}\right)}{\partial \ln m} \\
&= p_i \left(\frac{\partial q_i}{\partial \ln m} \frac{1}{m} \frac{q_i}{q_i} - q_i \frac{\partial m}{\partial \ln m} \frac{1}{m^2} \right) \\
&= p_i \left(\frac{\partial \ln q_i}{\partial \ln m} \frac{1}{m} q_i - \frac{q_i}{m} \right) \\
\mu_i &= e_i w_i - w_i \\
e_i &= \frac{\mu_i}{w_i} + 1 \qquad (11)
\end{aligned}
$$

Uncompensated (Marshallian) price elasticities:

$$
\begin{aligned}
\mu_{ij} &= \frac{\partial \left(\frac{p_i q_i}{m}\right)}{\partial \ln p_j} \\
&= \frac{1}{m} \left(\frac{\partial p_i}{\partial \ln p_j} q_i \frac{p_i}{p_i} + p_i \frac{\partial q_i}{\partial \ln p_j} \frac{q_i}{q_i} \right) \\
&= \frac{1}{m} \left(\frac{\partial \ln p_i}{\partial \ln p_j} p_i q_i + p_i \frac{\partial \ln q_i}{\partial \ln p_j} q_i \right) \\
&= w_i \left(\delta_{ij} + e^u_{ij} \right) \\
e^u_{ij} &= \frac{\mu_{ij}}{w_i} - \delta_{ij} \qquad (12)
\end{aligned}
$$

where δ_{ij} is the Kronecker delta

We get the compensated (Hicksian) price elasticities using the Slutsky equation:

$$
e^c_{ij} = e^u_{ij} + e_i w_j \qquad (13)
$$

4 Demographics and other household characteristics

We augment the QUAIDS model with other variables $\mathbf{x}$:

$$
w_i = \alpha_i + \sum_{j=1}^{n} \rho_{ij} x_j + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[\frac{m}{a(\mathbf{p})} \right] + \frac{\lambda_i}{b(\mathbf{p})} \left\{ \ln \left[\frac{m}{a(\mathbf{p})} \right] \right\}^2 \qquad (14)
$$

Adding-up, homogeneity and symmetry conditions are preserved if we impose:

$$\sum_{i=1} \rho_{ij} = 0 \tag{15}$$

5 Estimation

We estimate (14) using a non-linear seemingly unrelated regressions NL-SUR procedure. This is a two step procedure. In the first step parameters are estimated assuming independence of error terms across equations (therefore each equation can be estimated separately). From this first step an estimator of the variance-covariance matrix is obtained using estimated error terms. The second-step uses this estimator of the var-cov matrix to jointly estimate all parameters. All conditions discussed above are imposed, which implies that only $n-1$ equations are estimated. Notice that all parameters of the equation left out can be obtained using conditions (4) and (15)

6 Censoring

We follow the two-step estimation of a censored system procedure proposed by [9]. In this case the QUAIDS system is expressed as

$$w_i = \Phi(z'\theta_i) \left(\alpha_i + \sum_{j=1}^{n} \rho_{ij} x_j + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[\frac{m}{a(\mathbf{p})} \right] + \frac{\lambda_i}{b(\mathbf{p})} \left\{ \ln \left[\frac{m}{a(\mathbf{p})} \right] \right\}^2 \right) + \delta_i \phi(z'\theta_i) \tag{16}$$

where $\Phi(z'\theta_i)$ is the normal cumulative density function and $\phi(z'\theta_i)$ is the normal density function. z is a set of exogenous variables that explain the censoring mechanism.

6.1 Estimation

The estimation procedure is the following: (i) obtain probit estimates $\widehat{\theta}_i$ of θ_i using binary outcomes 0 if $w_i = 0$ and 1 if $w_i > 0$, (ii) calculate $\Phi(z'\widehat{\theta}_i)$ and $\phi(z'\widehat{\theta}_i)$ and estimate using NLSUR the following system

$$w_i = \Phi(z'\widehat{\theta}_i)\left(\alpha_i + \sum_{j=1}^{n}\rho_{ij}x_j + \sum_{j=1}^{n}\gamma_{ij}\ln p_j + \beta_i \ln\left[\frac{m}{a(\mathbf{p})}\right] + \frac{\lambda_i}{b(\mathbf{p})}\left\{\ln\left[\frac{m}{a(\mathbf{p})}\right]\right\}^2\right) + \delta_i\phi(z'\widehat{\theta}_i) \tag{17}$$

This estimation must be based on the full set of n equations as consumption shares do not add up to unity in general (See [10]). Homogeneity and symmetry conditions can be imposed as before. **(I am not sure that symmetry will hold imposing only (6) !!!)**

Also as the error terms in the system (17) are heteroskedastic the second step estimator is inefficient (See [9]). Standard errors can be estimated using bootstrap.

6.2 Elasticities

As before we differentiate equation (17) with respect to m and $\ln p_j$, respectively, to obtain

$$\mu_i^* \equiv \frac{\partial w_i}{\partial \ln m} = \Phi(z'\widehat{\theta}_i)\mu_i \tag{18}$$

and

$$\mu_{ij}^* \equiv \frac{\partial w_i}{\partial \ln p_j} = \Phi(z'\widehat{\theta}_i)\mu_{ij} \tag{19}$$

Both above derivations hold under the assumption that the vector z neither contains m nor any price. Expenditure and price elasticities have same expression as before

$$e_i = \frac{\mu_i^*}{w_i} + 1 \tag{20}$$

$$e_{ij}^u = \frac{\mu_{ij}^*}{w_i} - \delta_{ij} \tag{21}$$

$$e_{ij}^c = e_{ij}^u + e_i w_j \tag{22}$$

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