DISTANCE LEARNING

CONTENT

I. PRODUCER THEORY

SUGGESTED READING:

Varian, Hal R. *Intermediate Microeconomics*. W.W. Norton & Company, 7th Edition.
Notes from the fifth edition are available free of charge at :

http://people.ischool.berkeley.edu/~hal/Books/im-outline.pdf

II. CONSUMER THEORY

SUGGESTED READINGS:

Varian, Hal R. *Intermediate Microeconomics*. W.W. Norton & Company, 7th Edition.
Notes from the fifth edition are available free of charge at :

http://people.ischool.berkeley.edu/~hal/Books/im-outline.pdf

III. COMPETITIVE GENERAL EQUILIBRIUM MODEL

OBLIGATORY READING:

Reading_III-1.PDF : B. Decaluwé, A. Martens et L. Savard, (2001), *The Theoretical Framework or the Competitive General Equilibrium Model*, translation of Chapter 1 from La politique économique du développement et les modèles d'équilibre général calculable, Presses de l'Université de Montréal, Collection Universités francophones, Montréal.

SUGGESTED READINGS:

Reading_III-2.PDF: K. J. Arrow; G. Debreu (1954), *Existence of an Equilibrium for a Competitive Economy*, Econometrica, Vol. 22, No. 3, pp. 265-290.

Reading_III-3.PDF: S. Devarajan and S. Robinson, (2002), *The Impact of Computable General Equilibrium Models on Policy*, The World Bank and International Food Policy Research Institute (IFPRI).

Reading_III-4.PDF: J. B. Shoven and J. Whalley, (1984), *Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey*, Journal of Economic Literature, Vol. 22, No. 3, pp. 1007-1051.

Reading_III-5.PDF: I. S. Wing (2004), *Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis*, Center for Energy & Environmental Studies and Department of Geography & Environment Boston University and Joint Program on the Science & Policy of Global Change Massachusetts Institute of Technology, Technical Note No. 6

IV. SOCIAL ACCOUNTING MATRIX

OBLIGATORY READING:

Reading_IV-1.PDF : B. Decaluwé, A. Martens et L. Savard, (2001), *The Social Accounting Matrix: A Tool for Consistency*, translation of Chapter 5 from <u>La politique économique du développement et les modèles d'équilibre général calculable</u>, Presses de l'Université de Montréal, Collection Universités francophones, Montréal.

SUGGESTED READINGS:

Reading_IV-2.PDF: J. Round (2003), *Constructing SAMs for Development Policy Analysis: Lessons Learned and Challenges Ahead*.

Reading_IV-3.PDF: I. Fofana, A. Lemelin and J. Cockburn (2005), *Balancing a Social Accounting Matrix: Theory and Application*, Mimeo, CIRPEE et PEP, Université Laval.

Reading_IV-4.PDF: S. Robinson, A. Cattaneo, and M. El-Said (2001), *Updating and Estimating a Social Accounting Matrix Using Cross Entropy Methods*, Economic Systems Research, Vol. 13, No. 1, pp. 47-64.

V. PARAMETRIZATION AND INTRODUCTION TO GAMS

OBLIGATORY READING:

Reading_V-1.PDF: N. Annabi, J. Cockburn, and B. Decaluwé, (2006), *Functional Forms and Parametrization of CGE Models*, MPIA, Working Paper 2006-04.

Reading_V-2.PDF: Robichaud, V. (2009), *Introduction to GAMS Software, A Manuel for CGE Modelers*, Mimeo.

SUGGESTED READINGS:

Reading_V-3.PDF: Bruce A. McCarl (2000), *Course Materials from GAMS 2 Class Using GAMSIDE*, Agricultural Economics, Texas A&M.

I. PRODUCER THEORY

A FEW DEFINITIONS:

The firm's **production function** for a particular commodity, q, where $q = f(k,l)$ shows the maximum amount of the commodity that can be produced using alternative combinations of capital, k, and labor, l.

The **marginal physical product** of an input is the additional output that can be produced by employing one more unit of that input while holding all other inputs constant.

Marginal physical product of capital is:

$$MP_k = \frac{\partial q}{\partial k}$$

Similarly, marginal physical product of labor is:

$$MP_l = \frac{\partial q}{\partial l}$$

> *EXAMPLE : Let's assume that the production function of a given firm is*
>
> $$f(k,l) = 2k + kl + 7l^2$$
>
> *Then, marginal productivity of capital is:*
>
> $$MP_k = \frac{\partial f(k,l)}{\partial k} = 2 + l$$
>
> *And the marginal productivity of labor is:*
>
> $$MP_l = \frac{\partial f(k,l)}{\partial l} = k + 14l$$

Average physical productivity is the same as labor productivity. When it is said that a firm has experienced productivity increases, it means that output per unit of labor input has increased. Average productivity of labor is defined as:

$$AP = \frac{q}{l}$$

> *EXAMPLE: If we take the same production function as above, then the average productivity of labor is given by:*
>
> $$AP = \frac{2k + kl + 7l^2}{l} = \frac{2k}{l} + k + 7l$$

An **isoquant** records all combinations of capital and labor that can be used to produce a given quantity of output. The slope of an isoquant shows how one input can be traded for another while holding output constant.

The marginal rate of technical substitution (RTS) shows the rate at which labor can be substituted for capital while holding output constant along an isoquant.

To examine the shape of production function isoquants, it is useful to prove that the RTS (of *l* to *k*) is equal to the ratio of the marginal physical productivity of labor, MP_l, to the marginal physical productivity of capital MP_k:

$$RTS = \frac{MP_l}{MP_k}$$

> EXAMPLE: *Using our previous example, we can derive the RTS as follows:*
>
> $$RTS = \frac{k + 14l}{2 + l}$$

If the production function is given by $q = f(k, l)$ and if all inputs are multiplied by the same positive constant t (with t >1), then the **returns to scale** of the production function are:

Constant if $f(tk, tl) = tf(k, l) = tq$

Increasing if $f(tk, tl) > tf(k, l)$

Decreasing if $f(tk, tl) < tf(k, l)$

For the production function $q = f(k, l)$, the **elasticity of substitution** (σ) measures the proportionate change in k/l relative to the proportionate change in the RTS along an isoquant.

$$\sigma = \frac{\Delta(k/l)}{\Delta RTS} = \frac{d(k/l)}{dRTS} \cdot \frac{RTS}{k/l} = \frac{\delta \ln k/l}{\delta \ln RTS} = \frac{\delta \ln k/l}{\delta \ln f_l / f_k}$$

Along an isoquant, k/l and RTS move in the same direction, so the value of σ is always positive.

EXAMPLES OF PRODUCTION FUNCTIONS:

1. **The linear function:** $(\sigma = \infty)$

    $q = ak + bl$

    Returns to scale are constant:

    $$f(tk, tl) = atk + btl = t(ak + bl) = tq$$

    All isoquants for this production function are parallel straights lines with slope –b/a.

    Because RTS is constant along any straight line isoquant, the denominator in the definition of σ is equal to 0, meaning that σ is infinite.

This production function is very rare in practice, because it means that you could produce using only one of the inputs, depending on the inputs' prices.

2. **Leontief or fixed proportion** $(\sigma = 0)$

$q = \min(ak, bl)$ with a,b >0

Here, capital and labor must always be used in a fixed ratio. The isoquants for this production function are L-shaped. A firm with this production function will always operate along the ray where the ratio k/l is constant.

As k/l is constant, it is easy to see that $(\sigma = 0)$

You can find this function in many applications. Many machines require a certain number of people to run them, but any excess labor is superfluous.

Other example: if your firm produces a service of taxi. You have a car and a driver. Having more drivers would not be efficient; the same for a second car if you have one driver.

3. **The Cobb Douglas function:** $\sigma = 1$

$q = f(k,l) = Ak^a l^b$ with A, a and b > 0

Isoquants for this production function have a convex shape.

If a+b =1, the Cobb Douglas function exhibits constant returns to scale because output also increases by a factor of t.

If a+b>1, the function will have increasing return to scale

If a+b<1, the function will have decreasing return to scale

Indeed: $f(tk, tl) = A(tk)^a (tl)^b = At^{a+b} k^a l^b = t^{a+b} f(k,l)$

4. The CES production function:

$q = f(k,l) = \left[k^p + l^p \right]^{\chi/p}$

Where $0<p<1$ and $p \neq 0$, and $\chi \succ 0$

For $\chi \succ 1$, the function has increasing returns to scale, whereas for $\chi \prec 1$ it exhibits diminishing returns to scale.

What about σ ?

$$RTS = \frac{f_l}{f_k} = \frac{(\chi/p).q^{(\chi-p)/\chi}.pl^{p-1}}{(\chi/p).q^{(\chi-p)/\chi}.pk^{p-1}} = \left(\frac{l}{k} \right)^{p-1} = \left(\frac{k}{l} \right)^{1-p}$$

$$\sigma = \frac{\partial \ln(k/l)}{\partial \ln RTS} = \frac{1}{1-p}$$

If p=1, then we have the linear case.

If p=0, than we have the Cobb Douglas case

If p=$-\infty$, than we have the fixed-proportions.

The CES is often use with a distributional weight, $\beta(0 \leq \beta \leq 1)$, to indicate the relative significance of the inputs:

$$q = f(k,l) = \left[\beta k^p + (1-\beta)l^p\right]^{\chi/p}$$

ECONOMIC PROFITS AND COST MINIMIZATION

What is the cost for a firm to produce a given output?

$$C = wl + rk$$

If we assume that the firm only produces one unit of output, its total revenues are given by the price of its product (p) times its total output (q).

Economic profits are the difference between a firm's total revenues and its total costs:

$$\pi = pq - wl - rk$$

COST-MINIMIZING INPUT CHOICES

A firm seeks to minimize total costs given its technology q=f(k,l)=qo.

The Lagrangian is thus:

$$L = wl + rk + \lambda\left[qo - f(k,l)\right]$$

The first-order conditions for a constrained minimum are:

$$\frac{\partial L}{\partial l} = w - \lambda\frac{\partial f}{\partial l} = 0$$

$$\frac{\partial L}{\partial k} = r - \lambda\frac{\partial f}{\partial k} = 0$$

$$\frac{\partial L}{\partial \lambda} = qo - f(k,l) = 0$$

Or, using the first two equations,

$$\frac{w}{r} = \frac{\partial f / \partial l}{\partial f / \partial k} = RTS(lfork)$$

This says that the cost minimizing-firm should equate the RTS for the two inputs to the ratio of their prices.

EXAMPLE: *Assume that:*

$$q = f(k,l) = k^\alpha l^\beta$$

In that case, the Lagrangien expression for minimizing the cost of providing q0 is

$$L = rk + wl + \lambda\left(qo - k^\alpha l^\beta\right)$$

First order conditions for a minimum are:

$$\frac{\partial L}{\partial k} = r - \lambda\alpha k^{\alpha-1} l^\beta = 0$$

$$\frac{\partial L}{\partial l} = w - \lambda\beta k^\alpha l^{\beta-1} = 0$$

$$\frac{\partial L}{\partial \lambda} = qo - k^\alpha l^\beta = 0$$

Dividing the second of these by the first yields

$$\frac{w}{r} = \frac{\beta k^\alpha l^{\beta-1}}{\alpha k^{\alpha-1} l^\beta} = \frac{\beta}{\alpha} \cdot \frac{k}{l}$$

COST FUNCTIONS

The **total cost** function shows that, for any set of input costs and for any output level, the minimum total cost incurred by the firm is:

$$C = C(r,w,q)$$

The **average cost function** (AC) is found by computing total costs per unit of output:

$$AC(r,w,q) = \frac{C(r,w,q)}{q}$$

The **marginal cost function** (MC) is found by computing the change in total costs for a change in output produced:

$$MC(r,w,q) = \frac{\partial C(r,w,q)}{\partial q}$$

PROFIT MAXIMIZATION

Total revenues (R) are given by:

$$R(q) = p(q).q$$

The differences between revenues and costs (C) is called economic profits (π)

$$\pi(q) = p(q).q - C(q) = R(q) - C(q)$$

Maximizing profits implies:

$$\frac{d\pi}{dq} = \frac{dR}{dq} - \frac{dC}{dq} = 0$$

$$\frac{dR}{dq} = \frac{dC}{dq}$$

In other words, marginal revenue equals marginal cost.

To maximize economic profits, the firm should choose output for which marginal revenue is equal to marginal cost.

$$MR = \frac{dR}{dq} = \frac{dC}{dq} = MC$$

The condition we found is only necessary but not sufficient. For sufficiency, it is required that the marginal profit must be decreasing at the optimal level of q.

In mathematics terms, it means that:

$$\frac{d^2\pi}{dq^2} < 0$$

PROFIT MAXIMIZATION AND INPUT DEMAND

The firm's economic profits can be expressed as a function of only the inputs it employs:

$$\pi(q) = R(q) - C(q) = Pf(k,l) - (rk + wl)$$

The profit-maximizing firm's decision problem is to choose the appropriate levels of capital and labor input.

$$\frac{\partial \pi}{\partial k} = P\frac{\partial f}{\partial k} - r = 0$$

$$\frac{\partial \pi}{\partial l} = P\frac{\partial f}{\partial l} - w = 0$$

A profit maximizing firm should hire any input up to the point at which the input's marginal contribution to revenue is equal to the marginal cost of hiring the input.

The marginal revenue product is the extra revenue a firm receives when it employs one more unit of an input. When the firm is price taker (meaning it does not have any power on price, the firm cannot influence prices), the marginal revenue product of labor is equal to its market price.

SUBSTITUTION AND OUTPUT EFFECTS IN INPUT DEMAND

When the price of an input falls, two effects cause the quantity demanded of that input to rise:

1- the substitution effect causes any given output level to be produced using more of the output

2- the fall in costs causes more of the good to be sold, thereby creating an additional output effect that increases demand for the input.

For the rise in input price, both substitution and output effects cause the quantity of an input demanded to decline.

1. Fill the table below

$f(l,k)$	MP_l	MP_k	$RTS_{l,k}$
$l+2k$			
$a \cdot l + b \cdot k$			
$50 \cdot l \cdot k$			
$l^{0.25}k^{0.75}$			
$Cl^a k^b$			
$(l+2)(k+1)$			

2. Say for each production function whether it has constant, increasing or decreasing returns to scale.

a) $f(x_1, x_2) = x_1 + 2x_2$

b) $f(x_1, x_2) = 0{,}2 x_1 x_2^2$

c) $f(x_1, x_2) = x_1^{1/4} x_2^{3/4}$

3. Assume that your production function is given by

$$f(x, y) = x^{1/2} y^{3/2}$$

a) Write the expression of the marginal product of x.

b) Assume a little increase of x, y is constant. Does the marginal product of x increase, decrease or remain constant?

c) Do the same for y.

d) If y increases, what happens on the marginal product of x?

e) What is the RTS between y and x?

f) What can you say about returns to scale of this function?

4. Consider the cost function: $C(y) = 4y^2 + 16$

a) What is the average cost function?

b) What is the marginal cost function?

c) What is the level of production that minimizes the average cost?

d) What is the average cost function?

e) For what production level is the average cost equal to the marginal cost?

5. A firm in Los Angeles uses only one input. Its production function is : $f(x) = 4\sqrt{x}$ where x represents the number of input units. The output is sold at 100$ per unit, and the unitary cost of input is 50$.

a) Write an equation where the firm's profit depends on the output quantity.

b) What is the quantity of input that maximizes profit? What quantity of output maximizes the profit? How much is the maximal profit?

c) Suppose that a tax of 20$ is added on every unit of output while input price benefits from a subsidy of 10$. What are the profits now?

d) Suppose that instead of setting up this tax and this grant, firm's profit is taxed at 50%. Give the profit after tax depending on the input's quantity. What is the production that maximizes profit? What is the profit after tax?

6. Jean is a gardener. He thinks that the number of nice plants, *h*, depends on the quantity of light, *l*, and the quantity of water, *w*. Jean notices that plants need twice more light than water, and also that one more or one less unit does not have any effect. Thus, his production function is given by :

$$h = \min\{l, 2w\}$$

   a) Assume that Jean uses 1 unit of light. What is the minimal quantity of water he needs to produce a plant?

   b) Suppose that Jean wants to produce 4 plants. What are the minimal quantities of the two inputs he needs?

   c) What is the conditional demand function of light? What is the conditional demand function of water?

   d) If *a* is the cost of one light unit, and *b* the cost of a unit of water. What is Jean's cost function?

7. Flora is Jean's sister. She thinks that to get nice plants, you have to talk to them and give them fertilizer. Let *f* be the number of fertilizer's bag, and t the number of hours spent talking to plants.

   The number of plants is exactly equal to $h = t + 2f$.

   Assume that one fertilizer's bag costs *a* and a chat hour costs *b*.

   a) If Flora does not use fertilizers, how many hours will she have to talk to plants? If she does not talk at all, how many fertilizers' bags will she need for each plant?

   b) Suppose that b<a/2. In that case, if Flora wants to produce a plant, what will be cheaper for her: putting fertilizers or chatting?

   c) What is Flora's cost function?

II. CONSUMER THEORY

A FEW DEFINITIONS:

Individuals' preferences are assumed to be represented by a **utility function** of the form:

$$U(x_1, x_2, ..., x_n)$$

Where x_1, x_2, ...,x_n are the quantities of each n commodities that might be consumed in a period.

The **marginal utility** of a commodity is the utility gained from an increased consumption of this commodity. Mathematically, the marginal utility of X is given by:

$$MU_X = \frac{\partial U}{\partial X}$$

The **indifference curve** represents all the alternative combinations of x and y for which an individual is equally well off. Along an indifference curve, the utility is the same.

The slope of an indifference curve is negative showing that if the individual is forced to give up some y, he must be compensated by an additional amount of x to remain indifferent between the two bundles of goods. The negative of the slope of an indifference curve at some point is termed the **marginal rate of substitution** (MRS) at that point. That is:

$$MRS_{xy} = \frac{-dy}{dx} = \frac{MU_x}{MU_y}$$

EXAMPLES OF PREFERENCES:

Cobb Douglas utility:

$$U(x, y) = x^\alpha y^\beta$$

In general, the relative sizes of α and β indicate the relative importance of the two goods to the individual.

Perfect substitutes:

Here we will have linear indifference curves:

$$U(x, y) = \alpha x + \beta y$$

The indifference curves for this function are straight lines.

Because the MRS is constant and equal to α/β along the entire indifference curve, a person with these preferences would be willing to give up some amount of y to get one more x no matter how much x was being consumed.

Perfect complements:

$$U(x, y) = \min(\alpha x, \beta y)$$

Here we will have L-shaped indifference curves. These preferences would go for goods that go together like coffee and cream, peanut butter and jelly…These pairs of goods will be used in fixed proportion. If you want one cup of coffee with 2 spoons of cream, than you will want 2 cups of coffee and 4 spoons of cream. Only by choosing the goods together can utility be increased.

CES:

The three specific utility functions seen so far are special cases of the more general constant elasticity of substitution function (CES) which takes the form:

$$U(x, y) = A\left[\delta x^{-\rho} + (1-\delta)y^{-\rho}\right]^{\frac{-1}{\rho}}$$

Where $A \succ 0; 0 \prec \delta \prec 1; -1 \prec \rho \neq 0$

A Cobb-Douglas is a special case of the CES function for which $\rho = 0$, we have the Cobb Douglas situation.

To **maximize utility**, given a fixed amount of income to spend, a person will buy those quantities of goods that exhaust his total income and for which the psychic rate of trade off between the two goods (MRS) is equal to the rate at which the goods can be traded on for the other in the marketplace.

So, the person will equate the MRS to the ratio of the price of x to the price of y p_x / p_y.

First order condition for a maximum (FOC)

Assume that the individual has I \$ to allocate between 2 goods (x and y). If *a* is the price for x and *b* is the price for y, then the individual is constrained by:

$$ax + by \leq I$$

Slope of budget constraint= $\dfrac{-a}{b}$ = slope of indifference curve

Or, $\dfrac{a}{b} = MRS$

Indeed, for a utility maximum, all income should be spent and the MRS should equal the ratio of the prices of the goods.

Expenditure minimization:

This problem is clearly analogous to the primary utility-maximization problem, but the goals and constraints of the problem have been reversed.

The individual's dual expenditure minimization problem is to choose $x_1, x_2, ... x_n$ so as to minimize

total expenditure = E= $p_1 x_1 + p_2 x_2 + ... + p_n x_n$

Subject to constraint utility= $U = (x_1, x_2, ..., x_n)$

The individual's expenditure function shows the minimal expenditure necessary to achieve a given utility level for a particular set of prices. That is,

Minimal expenditures= $E(p_1, p_2, ..., p_n, U)$

What happens when there are changes in income or in the price of a good?

Changes in income:

As a person's purchasing power rises, it is natural to expect that the quantity of each good purchased will also increase. If both x and y increase as income increases, these goods are called normal goods.

For some goods however, the quantity chosen may decrease as income increases in some ranges. These goods are called inferior goods.

Changes in good prices:

When a prices changes, two different effects come into play:

- The substitution effect: even if the individual were to stay on the same indifference curve, consumption patterns would be allocated so as to equate the MRS to the new price ratio.

- The income effect: a change in price leads to a change in real income. The individual cannot stay in the same indifference curve and must move to a new one.

The utility-maximization hypothesis suggests that, for normal goods, a fall in the price of a good leads to an increase in quantity purchased because: the substitution effect causes more to be purchased as the individual moves along an indifference curve; and the income effect causes more to be purchased because the price decline has increased purchasing power, thereby permitting movement to higher indifference curve. When the price of a normal good rises, similar reasoning predicts a decline in the quantity purchased. For inferior goods, substitution and income effects work on opposite direction and there is no rule for total effect.

EXAMPLE: DETERMINING DEMAND FUNCTION

An individual spends his income R in buying two goods, X and Y of which prices are P_x and P_y.

Consumed quantities X and Y give to this consumer a utility function given by:

$$U(X,Y) = 2X^{1/2}Y^{1/2}$$

To determine demand function, we first have to write the maximization program. The individual dedicates all his income to the consumption of these 2 goods. His budget constraint is thus:

$$I = P_x X + P_y Y$$

Let's write the Lagrangian:

$$L = 2X^{1/2}Y^{1/2} + \lambda\left(I - P_x X - P_y Y\right)$$

We can now determine the FOC:

(1)
$$\frac{\partial L}{\partial X} = 2 \cdot \frac{1}{2} \cdot X^{-1/2} \cdot Y^{1/2} - \lambda P_x = 0$$

(2)
$$\frac{\partial L}{\partial Y} = 2 \cdot \frac{1}{2} \cdot X^{1/2} \cdot Y^{-1/2} - \lambda P_y = 0$$

(3)
$$\frac{\partial L}{\partial \lambda} = I - P_x X - P_y Y = 0$$

From (1) $\lambda = \dfrac{Y^{1/2}}{P_x X^{1/2}}$

From (2) $\lambda = \dfrac{X^{1/2}}{P_y Y^{1/2}}$

so: $\dfrac{Y^{1/2}}{P_x X^{1/2}} = \dfrac{X^{1/2}}{P_y Y^{1/2}} \Rightarrow P_x X = P_y Y$

Using this last identity into the budget constraint, we find the demand for each commodity:

$$I = P_x X + P_y Y = 2P_x X \Rightarrow X = \frac{I}{2P_x}$$

$$I = P_x X + P_y Y = 2P_y Y \Rightarrow Y = \frac{I}{2P_y}$$

DEMAND ELASTICITIES:

Price elasticity of demand $\left(e_{x,p_x}\right)$ measures the proportionate change in quantity demanded in response to a proportionate change in good's own price.

$$e_{x,p_x} = \frac{\Delta x / x}{\Delta p_x / p_x} = \frac{\Delta x}{\Delta p_x} \cdot \frac{p_x}{x} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x}$$

Income elasticity of demand $\left(e_{x,I}\right)$ measures the proportionate change in quantity demanded in response to a proportionate change in income.

$$e_{x,I} = \frac{\Delta x / x}{\Delta I / I} = \frac{\Delta x}{\Delta I} \cdot \frac{I}{x} = \frac{\partial x}{\partial I} \cdot \frac{I}{x}$$

Cross-price elasticity of demand $\left(e_{x,p_y}\right)$ measures the proportionate change in the quantity of x demanded in response to a proportionate change in the price of some other good (y).

$$e_{x,p_y} = \frac{\Delta x / x}{\Delta p_y / p_y} = \frac{\Delta x}{\Delta p_y} \cdot \frac{p_y}{x} = \frac{\partial x}{\partial p_y} \cdot \frac{p_y}{x}$$

1. Fill the table below

$U\ (x, y)$	MU_x	MU_y	MRS_{xy}
$2x + 3y$			
$2\sqrt{x} + y$			
$x \cdot y$			
$x^a y^b$			
$x^a + y^b$			

2. Let the utility function of a given consumer be:

$$U(X,Y) = 2X(Y+1)$$

    a. Derive the demand functions for each commodity.

    b. Assume that $P_x=P_y=1$. Determine the income R that allows reaching a utility of 32. Determine also corresponding consumptions.

    c. Now suppose that $P_x=2$ and $P_y=1$. Determine the new values of utility and consumptions when the income is the same as you found above.

3. Derive the demand functions for:

    a. A Cobb-Douglas utility function: $U(x, y) = x^\alpha y^\beta$

    b. A Leontief utility function $U(x, y) = \min(\alpha x, \beta y)$

    c. A CES utility function $U(x, y) = A\left[\delta x^{-\rho} + (1-\delta)y^{-\rho}\right]^{\frac{-1}{\rho}}$

III. COMPETITIVE GENERAL EQUILIBRIUM MODEL

EXERCISES

Assume that there are two (2) commodities produced in an economy. The production of each commodity *i* follows a Cobb-Douglas:

1. $XS_i = A_i LD_i^{\alpha_i} KD_i^{1-\alpha_i}$

Where XS_i, LD_i, KD_i represent output, labor demand and capital demand respectively, and A_i, α_i are parameters.

A.   Assuming that the corresponding prices of output, labor and capital are P_i, w, r derive the demand function for labor and capital.

2. $LD_i =$

3. $KD_i =$

B.   Let the utility function of the representative household also be a Cobb-Douglas function:

4. $U = B\, C_1^{\beta} C_2^{1-\beta}$

Where U, C_i represent utility and consumption of commodity i, and B, β are parameters. Assume further that household's income, Y, is simply the sum of return to labor and capital:

5. $Y = \sum_i \left(w\, LD_i + r\, KD_i \right)$

Assuming that consumer pays P_i for each commodity, derive the consumer demand functions:

6. $C_1 =$

7. $C_2 =$

C. Finally, assume that all markets are in equilibrium. Total labor supply, LS, is exogenous and the same goes for capital supply, KS. Derive equilibrium equation for each market:

8. $XS_i =$

9. $\overline{LS} =$

10. $\overline{KS} =$

D. How many variables are there in the simple general equilibrium model you just derived? How many are endogenous and how many are exogenous? Is your model square (that is, do you get the same number of equations and endogenous variables)?

E. What does the Walras' law imply?

IV. SOCIAL ACCOUNTING MATRIX

EXERCISES

Assume $i, j \in \{A, I, S\}$ be the set of productive activities and commodities (*A*: agriculture, *I*: industry, *S*: services) and $h \in \{HW, HC\}$ be the set of households (*HW*: labour endowed households (workers), *HC*: capital endowed households (capitalists)).

Fill the empty SAM on the next page using these variables. Only the shaded cells should be filled. As an example, dividends (DIV) should be at the intersection of row 4 and column 5 as firms pay dividends to the capital endowed households. It is thus an expenditure for the firms, and an income for households. Note that labor endowed households receive income from the labor market solely.

$CI_{i,j}$	Value of intermediate consumption of commodity *i* in activity *j*
DIV	Dividends paid by firms to households *HC*
$E_{i,h}$	Household *h*'s expenditure in commodity *i*
IT	Total investment
IV_i	Investment in good *i*
RK_F	Capital revenue paid to firms
RK_{HC}	Capital revenue paid to capital endowed households
RK_j	Return to capital from activity *j*
RK	Total capital revenue
SF	Firms' savings
SH_h	Household *h*'s savings
ST	Total savings
VX_j	Output of activity *j*
WL_j	Wage bill paid by activity *j*
WL	Total wage bill paid to workers
YF	Firms' income
YH_h	Household *h*'s income

Receipts →⟍ Expenses ↓	Factors of production		Agents			Productive activities			Acc.	TOTAL
	1.	2.	3.	4.	5.	6.	7.	8.	9.	(1 to 9)
1. Labor										
2. Capital										
3. Labor endowed household										
4. Capital endowed household					*DIV*					
5. Firms										
6. Agriculture										
7. Industry										
8. Services										
9. Accumulation										
TOTAL										

IV. PARAMETRIZATION

EXERCISE

Find below the mathematical specification for the model AUTA and a social accounting matrix (SAM). Assuming that all prices are equal to one, what are the parameters value that would be both consistent with the SAM and the mathematical specification?

Parameter v_j is calibrated as an example. Given equation 1: $v_j = \dfrac{VA_j}{XS_j}$

Using the SAM values we therefore get:

$$v_{AGR} = \frac{5,760 + 1,440}{9,000} = 0.80 \quad v_{IND} = \frac{7,560 + 11,340}{54,400} = 0.35 \quad v_{SER} = \frac{15,540 + 5,720}{30,700} = 0.69$$

$A_j =$

$\alpha_j =$

$aij_{i,j} =$

$\gamma_{i,h} =$

$io_j =$

$\lambda =$

$\mu_i =$

$\psi_h =$

A. EQUATIONS (50)

PRODUCTION

1. $VA_j = v_j \cdot XS_j$ — 3

2. $CI_j = io_j \cdot XS_j$ — 3

3. $VA_j = A_j \cdot LD_j^{\alpha_j} \cdot KD_j^{1-\alpha_j}$ — 3

4. $LD_j = \dfrac{\alpha_j \cdot PVA_j \cdot VA_j}{W}$ — 3

5. $DI_{i,j} = aij_{i,j} \cdot CI_j$ — 9

INCOME AND SAVINGS

6. $YH_{hw} = W \cdot \sum_j LD_j$ — 1

7. $YH_{hc} = \lambda \cdot \sum_j R_j \cdot KD_j + DIV$ — 1

8. $SH_h = \psi_h \cdot YH_h$ — 2

9. $YF = (1-\lambda) \cdot \sum_j R_j \cdot KD_j$ — 1

10. $SF = YF - DIV$ — 1

DEMAND

11. $C_{i,h} = \dfrac{\gamma_{i,h} \, YH_h}{P_i}$ — 6

12. $INV_i = \dfrac{\mu_i \, IT}{P_i}$ — 3

13. $DIT_i = \sum_j DI_{i,j}$ — 3

PRICES

14. $PVA_j = \dfrac{P_j \, XS_j - \sum_i P_i \, DI_{i,j}}{VA_j}$ — 3

15. $R_j = \dfrac{PVA_j \cdot VA_j - W \cdot LD_j}{KD_j}$ — 3

EQUILIBRIUM

16. $XS_i = DIT_i + \sum_h C_{i,h} + INV_i$ — 3

17. $LS = \sum_j LD_j$ — 1

18. $IT = \sum_h SH_h + SF$ — 1

B. ENDOGENOUS VARIABLES (50)

$C_{i,h}$:	Consumption of commodity i by type h households (volume)		6
CI_j :	Total intermediate consumption of industry j (volume)		3
$DI_{i,j}$:	Intermediate consumption of commodity i in industry j (volume)		9
DIT_i :	Total intermediate demand for commodity i (volume)		3
INV_i :	Final demand of commodity i for investment purposes (volume)		3
IT :	Total investment		1
LD_j :	Industry j demand for labour (volume)		3
P_i :	Price of commodity i		3
PVA_j :	Value added price for industry j		3
R_j :	Rate of return to capital from industry j		3
SF :	Firms' savings		1
SH_h :	Savings of type h household		2
VA_j :	Value added of industry j (volume)		3
W :	Wage rate		1
XS_j :	Output of industry j (volume)		3
YF :	Firms' income		1
YH_h :	Income of type h household		2

C. EXOGENOUS VARIABLES (5)

DIV :	Dividends		1
KD_j :	Industry j demand for capital (volume)		3
LS :	Total labour supply (volume)		1

D. PARAMETERS

A_j :	Scale coefficient (Cobb-Douglas production function)
$aij_{i,j}$:	Input-output coefficient
α_j :	Elasticity (Cobb-Douglas production function)
$\gamma_{i,h}$:	Share of the value of commodity i in total income of household h
io_j :	Technical coefficient (Leontief production function)
λ :	Share of capital income received by capitalists
μ_i :	Share of the value of commodity i in total investment
ψ_h :	Propensity to save
v_j :	Technical coefficient (Leontief production function)

E. SETS

$i, j \in I = \{AGR, IND, SER\}$	Industries and commodities (*AGR*: agriculture, *IND*: industry, *SER*: services)
$h \in H = \{HW, HC\}$	Households (*HW*: labour endowed households, *HC*: capital endowed households)

MODEL AUTA: A CLOSED ECONOMY WITHOUT GOVERNMENT

Expenses ↓ / Receipts →	FACTORS		AGENTS			PRODUCTIVE ACTIVITIES			ACC.	TOTAL
	1.	2.	3.	4.	5.	6.	7.	8.	9.	(1 to 9)
1. Labour						5,760	7,560	15,540		28,860
2. Capital						1,440	11,340	5,720		18,500
3. Labour endowed households	28,860									28,860
4. Capital endowed households		11,100			1,900					13,000
5. Firms		7,400								7,400
6. Agriculture			4,329	650		120	2,526.9	275.5	1,098.6	9,000
7. Industry			11,544	3,900		1,544	21,709.1	5,815.5	9,887.4	54,400
8. Services			10,101	5,850		136	11,264	3,349		30,700
9. Accumulation			2,886	2,600	5,500					10,986
TOTAL (1 to 9)	28,860	18,500	28,860	13,000	7,400	9,000	54,400	30,700	10,986	