Augmented Gravity Models:
(2) From the Gravity Equation to Gravity Models.

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Part I

The Gravity Equation

The gravity equation

For decades, since the 1960s, trade economists have used *the gravity equation* as a reduced form of a unidentified trade model.

$$\ln X_{ijt}^{k} = \ln G + \beta_1 \ln M_i + \beta_2 \ln M_j + \delta \ln d_{ij} + \epsilon_{ij} \tag{1}$$

- Great empirical success: $R^2 \simeq 0.6/0.7$;
- the fit is even better augmenting the gravity equation with different proxies for resistance to trade such as common language, borders, etc.;
- bilateral friction d_{ij} is not sufficient to capture opportunity costs: (1) the resistance to i's shipments on its other possible destinations; (2) resistance to shipments to j from j's other possible sources of supply.
- Early attempt: Remoteness $\frac{\sum_j d_{ij}}{Y_j}$.

Part II

Frictionless Gravity

Anderson's frictionless gravity

Smooth world: agents purchase goods in same proportions everywhere. Then:

$$\frac{X_{ij}}{E_j} = \frac{Y_i}{\sum_i Y_i} = \frac{Y_i}{Y} \tag{2}$$

- Multiply both sides by E_j, yielding frictionless gravity:

$$X_{ij} = \frac{Y_i E_j}{Y} = s_i b_j Y \tag{3}$$

where $b_j = \frac{E_j}{Y}$ and $s_i = \frac{Y_i}{Y}$.

Anderson's frictionless gravity

$$X_{ij} = \frac{Y_i E_j}{Y} = \underbrace{\frac{Y_i}{Y}}_{\text{Production share}} \cdot \underbrace{\frac{E_j}{Y}}_{\text{Consumption share}} \cdot Y = s_i b_j Y \qquad (4)$$

Implications:

1. Big producers have big market shares everywhere (e.g., $\frac{X_{ij}}{E_j} = s_i$);
2. small sellers are more open (i.e., openness) in the sense of trading more with the rest of the world (e.g.,

$$\frac{\sum_{i \neq j} X_{ij}}{E_j} = \frac{\sum_{i \neq j} Y_i}{Y} = \frac{Y - Y_j}{Y} = 1 - \frac{Y_j}{Y} = 1 - s_j;$$

   using $\sum_j E_j = \sum_i Y_i$, balanced trade for the world);
3. the world is more open the more similar in size and the more specialized the countries are;
4. the world is more open the greater the number of countries;
5. world openness rises with convergence under the simplifying assumption of balanced trade.

Implication (3): World openness

Given

$$X_{ij} = \frac{Y_i E_j}{Y} = \frac{Y_i}{Y} \cdot \frac{E_j}{Y} \cdot Y = s_i b_j Y \tag{5}$$

- Define world openness as the ratio of international shipments to total shipments, (e.g., $\frac{\sum_i \sum_j X_{ij}}{Y} = \sum_j b_j(1 - s_j) = 1 - \sum_j b_j s_j$);
- using standard properties of correlation

$$\rho_{sb} = \frac{cov(sb)}{\sqrt{\text{var}(s)\text{var}(b)}} = \frac{\frac{1}{N}\sum_{j=1}^{N} b_j s_j - \left(\frac{1}{N}\sum_j b_j\right)\left(\frac{1}{N}\sum_j s_j\right)}{\sqrt{\text{var}(s)\text{var}(b)}}$$

and $\sum_j s_j = \sum_j b_j = 1$:

$$\frac{\sum_i \sum_j X_{ij}}{Y} = 1 - \frac{1}{N} - N\ \rho_{sb}\sqrt{\text{var}(s)\text{var}(b)};$$

- $\text{var}(s)\text{var}(b)$ measures size dissimilarity
- ρ_{sb} is an inverse measure of specialization.

Implication (4): Openness and the number of countries

Given

$$O_w = \frac{\sum_i \sum_j X_{ij}}{Y} = 1 - \frac{1}{N} - N \underbrace{\rho_{sb}\sqrt{\text{var}(s)\text{var}(b)}}_{\leq 1};$$

- Differentiate w.r.t. N,

$$\frac{\partial O_w}{\partial N} = \frac{1}{N^2} - \rho_{sb}\sqrt{\text{var}(s)\text{var}(b)}.$$

- The result is ambiguous:
- World openness is increasing in the number of countries, if $\rho_{sb} \leq 0$ (complete specialization).

Part III

Structural Gravity

The gravity equation

After a generation of studies that micro-founded the gravity equation (Anderson, 1979; Bergstrand; 1989; Deardorff, 1998), the paper by Anderson and vanWincoop (2003) made a dramatic switch in the gravity literature.

From

$$X_{ij} = \frac{Y_i Y_j}{d_{ij}^{\delta}}, \tag{6}$$

to

$$X_{ij} = Y_i Y_j \left(\frac{d_{ij}}{\Pi_i P_j} \right)^{1-\sigma}, \tag{7}$$

The structural gravity equation can be (observational equivalence) from different classes of models:

- Differentiated Products in Demand, given supply and expenditure;
- Differentiated Productivity in Supply, with homogeneous demand;
- Discrete Choice Aggregation, in choice models à la McFadden (1973).

Anderson and vanWincoop (2003): Differentiated Demand

CES demand structure (final or intermediate)

$$X_{ij}^k = \left(\frac{\beta_i^k p_i^k t_{ij}^k}{P_j^k} \right)^{1-\sigma_k} E_j^k, \tag{8}$$

where

$$P_j^k = \left(\sum_i (\beta_i^k p_i^k t_{ij}^k)^{1-\sigma_k} \right)^{\frac{1}{1-\sigma_k}}, \tag{9}$$

- Using Market clearance condition, at end user prices

$$Y_i^k = \sum_j X_{ij}^k,$$

- and factor out $(\beta_i^k p_i^k)^{1-\sigma_k}$.

Anderson and vanWincoop (2003): Differentiated Demand

substitute for $(\beta_i^k p_i^k)^{1-\sigma_k}$, we get:

$$X_{ij}^k = \frac{Y_i E_j}{Y^k} \left(\frac{t_{ij}^k}{\Pi_i^k P_j^k} \right)^{1-\sigma_k}, \tag{10}$$

$$(\Pi_i^k)^{1-\sigma_k} = \sum_j \left(\frac{t_{ij}^k}{P_j^k} \right)^{1-\sigma_k} \frac{E_j^k}{Y^k}, \tag{11}$$

where

$$(P_j^k)^{1-\sigma_k} = \sum_i \left(\frac{t_{ij}^k}{\Pi_i^k} \right)^{1-\sigma_k} \frac{Y_i^k}{Y^k}, \tag{12}$$

- the system of equations (10)-(12) is the structural gravity model.
- Equation (10) is the trade flow equation.
- Π_i^k is the outward multilateral resistance (OMR) term,
- P_j^k is the inward multilateral resistance (IMR) term.

Anderson and vanWincoop (2003): Distance

From

$$X_{ij}^{k} = \frac{Y_i E_j}{Y^k} \left(\frac{t_{ij}^{k}}{\Pi_i^k P_j^k} \right)^{1-\sigma_{\boldsymbol{k}}}, \tag{13}$$

we can show that

- $\frac{X_{ij}^{k}}{Y_i} \frac{Y^k}{E_j}$ is the ratio of actual trade to frictionless trade;
- it depends on

$$t_{ij}^{k} = \exp(\delta_0 + \delta_1 d_{ij} + \delta_2 adj_{ij} + \cdots + \phi_i + \phi_j + \epsilon_{ij}),$$

- and on Π_i^k and P_j^k: the Multilateral resistance terms;
- where Multilateral resistance can be interpreted as the incidence of TFP frictions in the distribution of prices.

Part IV

Estimating Gravity

(After the break)