AGRODEP Technical Note 0020

December 2013

MIRAGRODEP 1.0: Documentation

David Laborde
Véronique Robichaud
Simla Tokgoz

Table of Contents

List of Figures

List of Tables

Abstract

MIRAGRODEP is a recursive-dynamic, multi-region, multi-sector computable general equilibrium model, devoted to trade and agricultural policy analysis. It is developed for AGRODEP and draws upon the MIRAGE model built by CEPII. It incorporates specific features such as foreign direct investment and runs with a tariff aggregation module that allows the user to capture the exclusion effects at a detailed level and the variance of tariffs. The model also includes a submodule allowing to test different closures for the public sector as well as the inefficiency of the tax collection system. Social Accounting Matrix (SAM) and trade data in MIRAGRODEP are based on the GTAP database. Additional sources such as MacMap are used for protection data.

1. Introduction

MIRAGRODEP is a Computable General Equilibrium (CGE) model based on MIRAGE (Modelling International Relations Under Applied General Equilibrium). It is a multi-region, multi-sector model, dynamically recursive[1] CGE model. MIRAGE was initially developed at CEPII and devoted to trade policy analysis. As opposed to a single country CGE model. A multi-country CGE model allows a detailed and consistent representation of Rest of the World. This way, international economic linkages are captured through the international trade of goods and foreign direct investment (FDI).

Social Accounting Matrix (SAM) and trade data in MIRAGRODEP is based on GTAP 8 (Narayanan and Walmsley (2012)). The GTAP Data Base is a fully documented, publicly available global data base which contains complete bilateral trade information, transport and protection linkages among 113 regions for all 57 GTAP commodities for 2004. For MIRAGRODEP, base year is 2007 and outlook period is from 2008 to 2025. For trade policy data, MAcMAP-HS6 has been used.

The objective of this AGRODEP Technical Note is two-folds. First, it aims to describe the mathematical structure[2] of and the economic hypothesis behind the MIRAGRODEP model, version 1.0 (available on AGRODEP website). In this version of MIRAGRODEP, the government is presented separately from the households and thus allows for a better understanding of the impact of shocks on the private and the public sectors distinctly. Furthermore, MIRAGRODEP models are simplified versions of the MIRAGE model (Decreux and Valin, 2007). As the MIRAGE model has been fully documented, parts of this document are extracts from Decreux and Valin (2007). The second aim of the document is to explain how practically the GAMS code can be used for running MIRAGRODEP for reference scenario (baseline) and for scenario simulations.

The document is organized as follows. In Section 2, we present the main pillars of the model structure, with a summary of equations and variables mapped to their counterparts in GAMS code. The model structure is summarized in Section 3.

2. Model Structure

2.1 Dimensions and sets

The MIRAGRODEP model distinguishes multiple sectors (or activities, industries) each of them producing one single commodity (or good, product). Sectors and commodities are referred to using indices i or j, both representing the exact same elements. The subset *Transport* refers to the transportation commodities and sectors.

[1] Dynamically recursive models do not include expectation of value of variables in future periods in the model. Plus, value of variable X at the end of period t is the initial value of variable X at the beginning of period t+1.

[2] For a comprehensive review of the functional forms commonly used in CGE models, please refer to Femenia (2012).

MIRAGRODEP is a global dynamic model. Each variable is thus indexed in time (index t) and by region using indices r (origin country), s (destination country), *rr* and *ss*, which all correspond to the same elements.

Set *f* refers to the five (5) factors of production: skilled labor (index *SkLab*), unskilled labor (*UnSkLab*), natural resources (*NatlRes*), capital (*Capital*) and land (*Land*). As will be discussed below, it is assumed that unskilled workers are not perfectly mobile across sectors of production. Hence, sectors are grouped according to the area, rural (*L1*) or urban (*L2*), both elements being included in set *Ltype*.

2.2 Production

The production in each sector and in each region follows the nested structure depicted in Figure 1 below. At the top level, total output $Y_{j,r,t}$ is a Leontief of total value added, $VA_{j,r,t}$, and of total intermediate consumption, $CNTER_{j,r,t}$. In other words, there are no substitution possibilities between the two aggregated inputs, they are used in perfect complementarity, and thus their shares in total production are constant.

Mathematically:

$$Y_{j,r,t} = a_{j,r}^{VA} VA_{j,r,t} \tag{1}$$

$$Y_{j,r,t} = a_{j,r}^{CNTER} CNTER_{j,r,t} \tag{2}$$

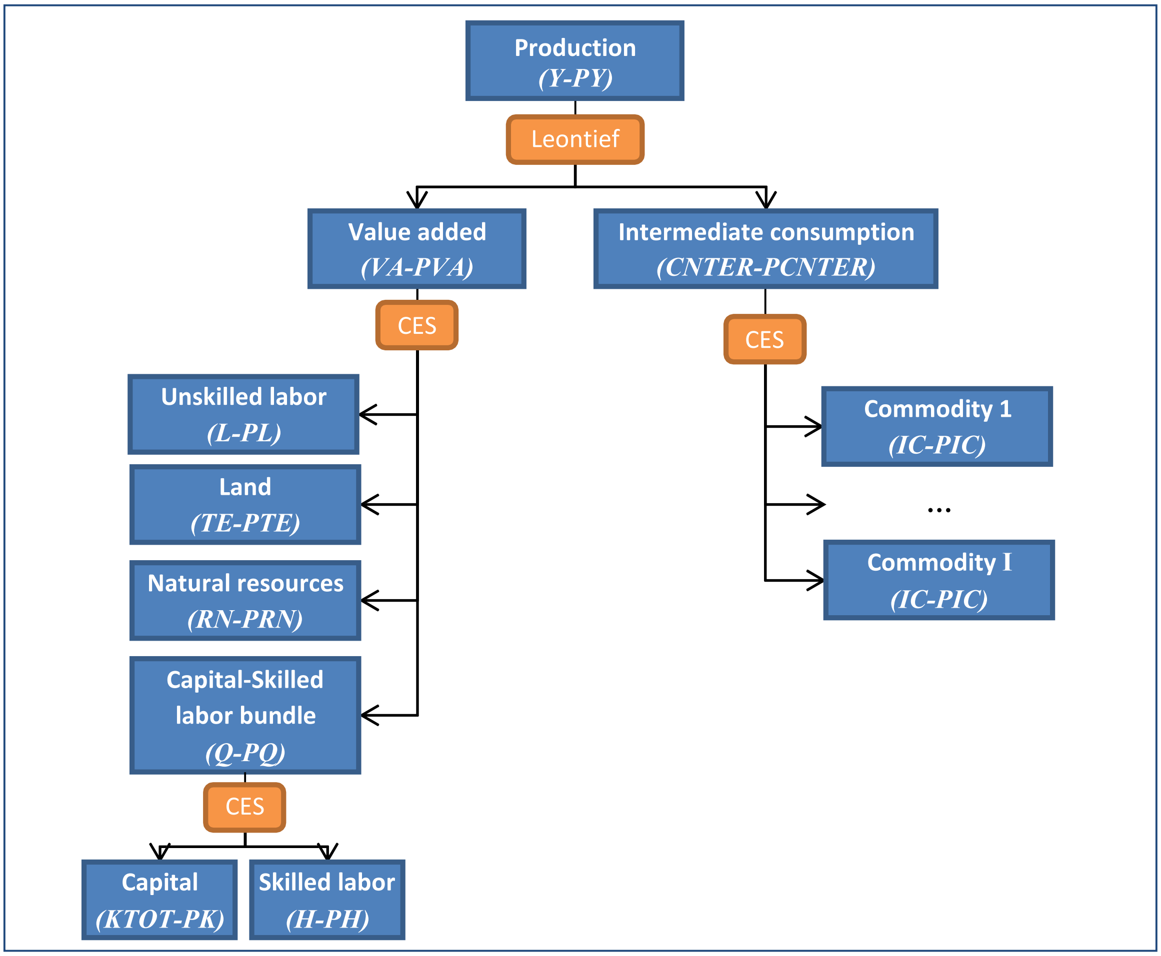
with

$a_{j,r}^{VA}$	Value added scale coefficient
$a_{j,r}^{CNTER}$	Total intermediate consumption scale coefficient

Hence, the producer price of output, $PY_{j,r,t}$, is a weighted sum of the price of value added, $PVA_{j,r,t}$, and of that of total intermediate consumption, $PCNTER_{j,r,t}$.

$$PY_{j,r,t}\, Y_{j,r,t} = PVA_{j,r,t}\, VA_{j,r,t} + PCNTER_{j,r,t}\, CNTER_{j,r,t} \tag{3}$$

Figure 1. Nested production function[3]



At the second level, on the value added side, total value added is a combination of unskilled labor, $L_{j,r,t}$, land, $TE_{j,r,t}$, natural resources, $RN_{j,r,t}$, and capital-skilled labor bundle, $Q_{j,r,t}$. It is assumed that these inputs are imperfect substitutes for one another, which is represented through the use of a constant elasticity of substitution (CES) function[4]. The representative firm minimizes its costs subject to the CES aggregator, which yield the following first order conditions:

$$L_{j,r,t} = a_{j,r}^{L} \, VA_{j,r,t} \, PGF_{r,t}{}^{\sigma_j^{VA}-1} \left(\frac{PVA_{j,r,t}}{PL_{j,r,t}}\right)^{\sigma_j^{VA}} \quad (4)$$

$$TE_{j,r,t} = a_{j,r}^{TE} \cdot VA_{j,r,t} \cdot PGF_{r,t}{}^{\sigma_j^{VA}-1} \cdot \left(\frac{PVA_{j,r,t}}{PTE_{j,r,t}}\right)^{\sigma_j^{VA}} \quad (5)$$

[3] The acronyms for the volume followed by its corresponding price appear in brackets.
[4] It might be worth noting that some parameters are solely indexed in *j*. It is the case, for example, for the elasticity used in the value added functions (σ^{VA}). This specification implies that the same parameter is used for all regions, but that it differs from one sector to the other.

$$RN_{j,r,t} = a^{RN}_{j,r} \cdot VA_{j,r,t} \cdot PGF_{r,t}{}^{\sigma_j^{VA}-1} \left(\frac{PVA_{j,r,t}}{PRN_{j,r,t}}\right)^{\sigma_j^{VA}} \quad (6)$$

$$Q_{j,r,t} = a^{Q}_{j,r} \cdot VA_{j,r,t} \cdot PGF_{r,t}{}^{\sigma_j^{VA}-1} \cdot \left(\frac{PVA_{j,r,t}}{PQ_{j,r,t}}\right)^{\sigma_j^{VA}} \quad (7)$$

with

$a^{L}_{j,r}$	Unskilled labor coefficient
$a^{TE}_{j,r}$	Land coefficient
$a^{RN}_{j,r}$	Natural resources coefficient
$a^{Q}_{j,r}$	Capital-skilled labor aggregate coefficient
σ_j^{VA}	Value added elasticity
$PGF_{r,t}$	Total factor productivity

It follows that the price of value added is a weighted sum of the price of unskilled labor, $PL_{j,r,t}$ the price of land, $PTE_{j,r,t}$ the price of natural resources, $PRN_{j,r,t}$, and the aggregated price of capital and skilled workers, $PQ_{j,r,t}$.

$$PVA_{j,r,t}\, VA_{j,r,t} = PL_{j,r,t}\, L_{j,r,t} + PTE_{j,r,t}\, TE_{j,r,t} + PRN_{j,r,t}\, RN_{j,r,t} + PQ_{j,r,t}\, Q_{j,r,t} \quad (8)$$

The price paid by the producer for each factor differs from the one received by the households by the amount of taxes, which can be negative in the cases where factors are subsidized. The model also distinguishes ad valorem taxes from taxes that are applied on volume. Hence:

$$PL_{j,r,t} = WLt_{Ltype,r,t} \left(1 + taxf^{VAL}_{UnSkLab,j,r,t}\right) + PIndC_{r,t}\, taxf^{VOL}_{UnSkLab,j,r,t} \quad (9)$$

$$PTE_{j,r,t} = WTE_{j,r,t} \left(1 + taxf^{VAL}_{Land,j,r,t}\right) + PIndC_{r,t}\, taxf^{VOL}_{Land,j,r,t} \quad (10)$$

$$PRN_{j,r,t} = WRN_{j,r,t} \left(1 + taxf^{VAL}_{NatlRes,j,r,t}\right) + PIndC_{r,t}\, taxf^{VOL}_{NatlRes,j,r,t} \quad (11)$$

with

$WLt_{Ltype,r,t}$	Rate of return to unskilled labor (net of taxes)
$WTE_{j,r,t}$	Rate of return to land (net of taxes)
$WRN_{j,r,t}$	Rate of natural resources (net of taxes)
$PIndC_{r,t}$	Consumer price index
$taxf^{VAL}_{f,j,r,t}$	Rate of factor-based taxes (ad valorem)
$taxf^{VOL}_{f,j,r,t}$	Rate of factor-based taxes (on volume)

At the bottom level, on the value added side, capital, $KTOT_{j,r,t}$ and skilled labor, $H_{j,r,t}$, are combined through a CES function, once again to represent the imperfect substitutability between the two factors of production. Minimization of production costs subject to the CES aggregator gives the following demand functions:

$$H_{j,r,t} = a_{j,r}^{H} Q_{j,r,t} \left(\frac{PQ_{j,r,t}}{PH_{j,r,t}}\right)^{\sigma_j^{CAP}} \quad (12)$$

$$KTOT_{j,r,t} = a_{j,r}^{K} Q_{j,r,t} \left(\frac{PQ_{j,r,t}}{PK_{j,r,t}}\right)^{\sigma_j^{CAP}} \quad (13)$$

with

$a_{j,r}^{H}$	Skilled labor coefficient
$a_{j,r}^{K}$	Capital coefficient
σ_j^{CAP}	Capital-skilled labor elasticity

The price of the capital-skilled labor bundle is thus a weighted sum of the rental rate of capital, $PK_{j,r,t}$, and of the price of skilled labor, $PH_{j,r,t}$.

$$PQ_{j,r,t}\, Q_{j,r,t} = PK_{j,r,t}\, KTOT_{j,r,t} + PH_{j,r,t}\, H_{j,r,t} \quad (14)$$

Again, the prices paid for the factors of production differ from the ones received by households as there are taxes levied on each of them.

$$PH_{j,r,t} = WH_{r,t} \left(1 + taxf_{SkLab,j,r,t}^{VAL}\right) + PIndC_{r,t}\, taxf_{SkLab,j,r,t}^{VOL} \quad (15)$$

$$PK_{j,r,t} = WK_{j,r,t} \left(1 + taxf_{Capital,j,r,t}^{VAL}\right) + PIndC_{r,t}\, taxf_{Capital,j,r,t}^{VOL} \quad (16)$$

with

$WH_{r,t}$	Rate of return to skilled labor (net of taxes)
$WK_{j,r,t}$	Rate of return to capital (net of taxes)

On the intermediate consumption side, the commodities (index *i*) used in the production process are assumed to be imperfect substitutes. Once again, a CES function is used to represent this imperfect substitutability, and cost minimization yields the demand for each input, $IC_{i,j,r,t}$:

$$IC_{i,j,r,t} = a_{i,j,r}^{IC} CNTER_{j,r,t} \left(\frac{PCNTER_{j,r,t}}{PIC_{i,j,r,t}}\right)^{\sigma^{IC}} \quad (17)$$

with

$a_{i,j,r}^{IC}$	Intermediate consumption scale coefficient

σ^{IC} Intermediate consumption elasticity

The price of total intermediate consumption is a weighted sum of the price paid for each commodity, $PIC_{i,j,r,t}$.

$$PCNTER_{j,r,t}\ CNTER_{j,r,t} = \sum_i PIC_{i,j,r,t}\ IC_{i,j,r,t} \tag{18}$$

The price of each input is subject to taxes, $taxicc_{i,j,r,t}$, and thus differ from the price received by producers $PDEMTOT_{i,r,t}$.

$$PIC_{i,j,r,t} = PDEMTOT_{i,r,t}\ (1 + taxicc_{i,j,r,t}) \tag{19}$$

2.3 Income and savings

2.3.1 Households

Households are assumed to be homogenous and they own all factors of production. They, hence, receive all the payments made to factors of production. They also receive transfers from the government, which are indexed to take into account population growth and the evolution of the price index.

$$REVH_{r,t} = \sum_j \left\{ WRN_{j,r,t}\ RN_{j,r,t} + WTE_{j,r,t}\ TE_{j,r,t} + WH_{r,t}\ H_{j,r,t} + \sum_{Ltype} WLt_{Ltype,r,t}\ L_{j,r,t} + \sum_s WK_{j,s,t}\ K_{j,r,s,t} \right\} + Pop^{ag}_{totpop,r,t}\ TRH_{r,t}\ PIndC_{r,t} \tag{20}$$

with

$REVH_{r,t}$	Households' income
$TRH_{r,t}$	Public transfers to households
$Pop^{ag}_{totpop,r,t}$	Population

Households savings, $SAVH_{r,t}$, are a fixed proportion epa_r of their income net of indirect taxes, $RECDIR_{r,t}$, and the rest of their income is dedicated to consumption budget, $BUDH_{r,t}$.

$$SAVH_{r,t} = epa_r\ (REVH_{r,t} - RECDIR_{r,t}) \tag{21}$$

$$BUDH_{r,t} = REVH_{r,t} - SAVH_{r,t} - RECDIR_{r,t} \tag{22}$$

2.3.2 Government

The income of the government, $REVG_{r,t}$, consists of taxes collected on production, $RECPROD_{i,r,t}$, on factors of production, $RECFAC_{i,r,t}$, on exports, $RECEXP_{i,r,t}$, on imports, $RECDD_{i,r,t}$, on consumption, $RECCONS_{i,r,t}$, and households' income, $RECDIR_{r,t}$.

$$REVG_{r,t} = \sum_i \{RECPROD_{i,r,t} + RECFAC_{i,r,t} + RECEXP_{i,r,t} + RECDD_{i,r,t} + RECCONS_{i,r,t}\} + RECDIR_{r,t} \quad (23)$$

Taxes on production are collected on the value of output of each activity. It is important to note that tax rates should be considered as *net* rates, that is taxes net of subsidy. Hence, all tax rates can be either positive or negative.

$$RECPROD_{i,r,t} = taxP_{i,r,t}\, PY_{i,r,t}\, Y_{i,r,t} \quad (24)$$

with

$taxP_{i,r,t}$ Production tax rate

Receipt from taxes on factors of production is the sum of volume and value taxes on each factor.

$$\begin{aligned} RECFAC_{j,r,t} = {} & PIndC_{r,t} \Big(taxf^{VOL}_{Land,j,r,t}\, TE_{j,r,t} + taxf^{VOL}_{NatlRes,j,r,t}\, RN_{j,r,t} + taxf^{VOL}_{SkLab,j,r,t}\, H_{j,r,t} \\ & + taxf^{VOL}_{UnSkLab,j,r,t}\, L_{j,r,t} + taxf^{VOL}_{Capital,j,r,t}\, KTOT_{j,r,t} \Big) \\ & + taxf^{VAL}_{Land,j,r,t}\, WTE_{j,r,t}\, TE_{j,r,t} + taxf^{VAL}_{NatlRes,j,r,t}\, WRN_{j,r,t}\, RN_{j,r,t} \\ & + taxf^{VAL}_{SkLab,j,r,t}\, WH_{r,t}\, H_{j,r,t} \\ & + taxf^{VAL}_{UnSkLab,j,r,t} \sum_{Ltype} WLt_{Ltype,r,t}\, L_{j,r,t} + taxf^{VAL}_{Capital,j,r,t}\, WK_{j,r,t}\, KTOT_{j,r,t} \end{aligned} \quad (25)$$

Exports may be subject to three taxes: taxes on production, $taxP_{i,r,t}$, regular taxes on exports, $taxEXP_{i,r,s,t}$, and export tax equivalent of multi-fiber arrangement quota premium, $taxAMF_{i,r,s,t}$.

$$RECEXP_{i,r,t} = PY_{i,r,t}(1 + taxP_{i,r,t}) \sum_s (taxEXP_{i,r,s,t} + taxAMF_{i,r,s,t}) TRADE_{i,r,s,t} \quad (26)$$

with

$TRADE_{i,r,s,t}$ Exports of commodity i from country r to country s

Duties, $DD_{i,s,r,t}$, are collected on imports evaluated at the CIF price, $PCIF_{i,s,r,t}$.

$$RECDD_{i,r,t} = \sum_s DD_{i,s,r,t}\, PCIF_{i,s,r,t}\, TRADE_{i,s,r,t} \tag{27}$$

Taxes are levied on households' consumption, $CH_{i,r,t}$, government current expenditure on goods and services, $CG_{i,r,t}$, on commodities sold for investment purposes, $KG_{i,r,t}$, and on intermediate consumption, $IC_{i,j,r,t}$. Each buyer faces a specific tax rate, respectively , $taxcc_{i,r,t}$, $taxgc_{i,r,t}$, $taxkgc_{i,r,t}$, and $taxicc_{i,j,r,t}$.

$$RECCONS_{i,r,t} = PDEMTOT_{i,r,t} \left\{ taxcc_{i,r,t}\, CH_{i,r,t} + taxgc_{i,r,t}\, CG_{i,r,t} + taxkgc_{i,r,t}\, KG_{i,r,t} + \sum_j taxicc_{i,j,r,t}\, IC_{i,j,r,t} \right\} \tag{28}$$

Finally, the government collects direct taxes on households' income:

$$RECDIR_{r,t} = taxdir_{r,t}\, REVH_{r,t} \tag{29}$$

Government savings, $SAVG_{r,t}$, are assumed to be a fixed proportion, $PUBSOLD_r$, of GDP at market prices, $GDPMP_{r,t}$. Finally, the budget allocated to public current expenditure on goods and services, $BUDG_{r,t}$, is determined residually.

$$SAVG_{r,t} = PUBSOLD_r\, GDPMP_{r,t} \tag{30}$$

$$BUDG_{r,t} = REVG_{r,t} - SAVG_{r,t} - Pop^{ag}_{totpop,r,t}\, TRH_{r,t}\, PIndC_{r,t} \tag{31}$$

2.4 Demand

Domestic absorption of each commodity, $DEMTOT_{i,r,t}$, is the sum of consumer demand, $CH_{i,r,t}$, demand from public administrations, $CG_{i,r,t}$, intermediate demand, $IC_{i,j,r,t}$, and demand for investment purposes, $KG_{i,r,t}$.

$$DEMTOT_{i,r,t} = CH_{i,r,t} + CG_{i,r,t} + \sum_j IC_{i,j,r,t} + KG_{i,r,t} \tag{32}$$

2.4.1 Private demand

Households' demand is characterized by a LES-CES (Linear Expenditure System - Constant Elasticity of Substitution) specification. This specific utility function allows the evolution of the demand structure of each region to be accounted for as its income level changes. Additionally, the elasticity of substitution is constant only among the sectoral consumptions over and above a minimum level. The minimal level of consumption can vary across region (e.g. developing versus developed country).

$$CH_{i,r,t} = Pop^{ag}_{totpop,r,t} \left(cmin_{i,r} + a^C_{i,r} \, AUX_{r,t} \left(\frac{P_{r,t}}{PC_{i,r,t}} \right)^{\sigma^C_r} \right) \tag{33}$$

with

$cmin_{i,r}$	Minimal consumption of commodity i (per capita)
$a^C_{i,r}$	Household consumption coefficient
$AUX_{r,t}$	Utility
$P_{r,t}$	Shadow price of utility
$PC_{i,r,t}$	Price of final private consumption
σ^C_r	Households' consumption elasticity of substitution

Households maximize their utility subject to their consumption budget, $BUDH_{r,t}$, from which one can derive the shadow price of utility, $P_{r,t}$.

$$BUDH_{r,t} = \sum_i PC_{i,r,t} \, CH_{i,r,t} \tag{34}$$

$$P_{r,t} \, AUX_{r,t} = \sum_i PC_{i,r,t} \left(\frac{CH_{i,r,t}}{Pop^{ag}_{totpop,r,t}} - cmin_{i,r} \right) \tag{35}$$

The price paid by household for each commodity, $PC_{i,r,t}$, differs from the one received by the suppliers, $PDEMTOT_{i,r,t}$, by the amount of taxes collected, $taxcc_{i,r,t}$.

$$PC_{i,r,t} = PDEMTOT_{i,r,t} \, (1 + taxcc_{i,r,t}) \tag{36}$$

Finally, the consumer price index, $PIndC_{r,t}$, is a Fisher index.

$$PindC_{r,t} = \sqrt{ \left[\frac{\sum_i PC_{i,r,t} \, CH^O_{i,r}}{\sum_i PC^O_{i,r} \, CH^O_{i,r}} \right] \left[\frac{\sum_i PC_{i,r,t} \, CH_{i,r,t}}{\sum_i PC^O_{i,r} \, CH_{i,r,t}} \right] } \tag{37}$$

with

$CH^O_{i,r}$	Benchmark value of households' consumption
$PC^O_{i,r}$	Benchmark value of final private consumption

2.4.2 Public demand

Government spending on each commodity is a fixed share, $\alpha_{i,r}^{G}$, of total public expenditure in goods and services, $BUDG_{r,t}$, and government purchases are subject to taxes, $taxgc_{i,r,t}$.

$$PCG_{i,r,t}CG_{i,r,t} = \alpha_{i,r}^{G}\, BUDG_{r,t} \tag{38}$$

$$PCG_{i,r,t} = PDEMTOT_{i,r,t}\left(1 + taxgc_{i,r,t}\right) \tag{39}$$

with

$PCG_{i,r,t}$	Price of final public consumption

2.4.3 Demand for investment purposes

Finally, demand for investment purposes, $KG_{i,r,t}$, is characterized by a CES function. Cost minimization subject to the CES aggregator yields the following demand function:

$$KG_{i,r,t} = a_{i,r}^{KG}\, INVTOT_{r,t}\left(\frac{PINVTOT_{r,t}}{PKG_{i,r,t}}\right)^{\sigma^{KG}} \tag{40}$$

with

$a_{i,r}^{KG}$	Capital good scale coefficient
$INVTOT_{r,t}$	Total investment
$PINVTOT_{r,t}$	Price of investment
$PKG_{i,r,t}$	Price of capital good consumption
σ^{KG}	Capital good elasticity

The aggregated price of capital, $PINVTOT_{r,t}$, is thus a weighted sum of the price paid for each commodity, $PKG_{i,r,t}$.

$$PINVTOT_{r,t}\, INVTOT_{r,t} = \sum_{i} PKG_{i,r,t}\, KG_{i,r,t} \tag{41}$$

Again, the price paid by the purchaser differs from the one received by the seller, as taxes apply.

$$PKG_{i,r,t} = PDEMTOT_{i,r,t}\,(1 + taxkgc_{i,r,t}) \tag{42}$$

2.4.4 Demand by geographic origin

MIRAGRODEP is a bilateral trade model consistent with the Armington assumption: commodities are assumed to be heterogeneous according to their origin, and thus, imperfect substitutes for one another (Armington 1969). Nested CES functions are used to reflect preferences among varieties originating from different countries. Therefore, countries can export and import the same product at the same time due to consumer preferences for different varieties. The price transmission between domestic and international market is imperfect and highly dependent on the choice of the CES trade elasticities and the initial share of trade.

At the top level, total demand, $DEMTOT_{i,r,t}$, combines aggregated imports, $M_{i,r,t}$, and local production, $D_{i,r,t}$, through a CES function. From cost minimization subject to the CES aggregator, the following demand functions can be derived:

$$D_{i,r,t} = a_{i,r}^{D} \, DEMTOT_{i,r,t} \left(\frac{PDEMTOT_{i,r,t}}{PD_{i,r,t}}\right)^{\sigma_i^{ARM}} \qquad (43)$$

$$M_{i,r,t} = a_{i,r}^{M} \, DEMTOT_{i,r,t} \left(\frac{PDEMTOT_{i,r,t}}{PM_{i,r,t}}\right)^{\sigma_i^{ARM}} \qquad (44)$$

with

$a_{i,r}^{D}$	Local demand scale coefficient
$a_{i,r}^{M}$	Total import demand scale coefficient
σ_i^{ARM}	Armington elasticity
$PD_{i,r,t}$	Price of demand for domestic commodity
$PM_{i,r,t}$	Aggregated price of imports

Consequently, the price of the aggregated commodity, $PDEMTOT_{i,r,t}$, is a weighted sum of aggregated imports, $PM_{i,r,t}$, and of the price of the domestically produced commodity, $PD_{i,r,t}$, which differs from the amount received by the producer, $PY_{i,r,t}$, since taxes, $taxP_{i,r,t}$, apply.

$$PDEMTOT_{i,r,t} \, DEMTOT_{i,r,t} = PD_{i,r,t} \, D_{i,r,t} + PM_{i,r,t} \, M_{i,r,t} \qquad (45)$$

$$PD_{i,r,t} = PY_{i,r,t} \, (1 + taxP_{i,r,t}) \qquad (46)$$

At the second level, total imports, $M_{i,r,t}$, are a CES combination of imports from the different trading partners, $DEMA_{i,s,r,t}$. Cost minimization under the CES aggregation constraint leads to the following demand function:

$$DEMA_{i,s,r,t} = a_{i,s,r}^{IMP} \, M_{i,r,t} \left(\frac{PM_{i,r,t}}{PDEMA_{i,s,r,t}}\right)^{\sigma_i^{IMP}} \qquad (47)$$

with

$a_{i,s,r}^{IMP}$	Import demand scale coefficient
σ_i^{IMP}	Import elasticity
$PDEMA_{i,s,r,t}$	Price of bilateral trade

This specification implies that the price of aggregated imports is a weighted sum of the price paid to the different partners. The price paid by the purchaser differs from the CIF price as import duties, $DD^{A}_{i,s,r,t}$ apply.

$$PM_{i,r,t}\, M_{i,r,t} = \sum_{s} PDEMA_{i,s,r,t}\, DEMA_{i,s,r,t} \tag{48}$$

$$PDEMA_{i,s,r,t} = PCIF_{i,s,r,t}\left(1 + DD^{A}_{i,s,r,t}\right) \tag{49}$$

And the CIF price is determined by the production costs, on which taxes apply, plus the transportation costs.

$$PCIF_{i,s,r,t} = PY_{i,s,t}\,(1 + taxEXP_{i,s,r,t} + taxAMF_{i,s,r,t})\,(1 + taxP_{i,s,t}) + MUO_{i,s,r}\, PTr_{i,s,r,t} \tag{50}$$

with

$PTr_{i,s,r,t}$	Price of transportation per commodity exported
$MUO_{i,s,r}$	Transport coefficient

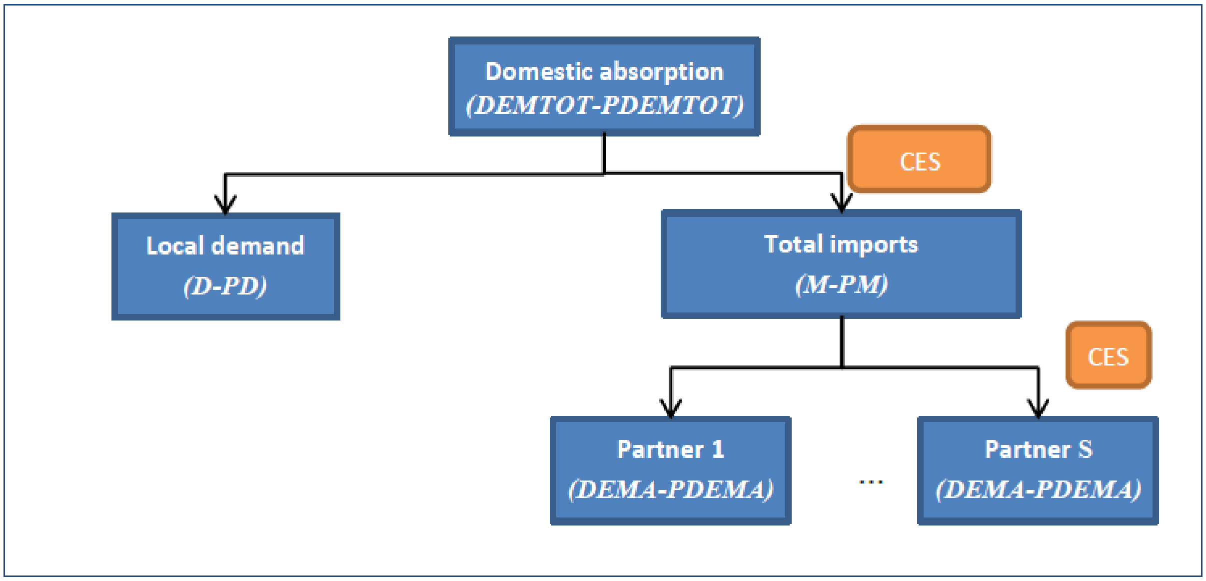
Following the consistent aggregator methodology as defined in(Laborde, Martin, and van der Mensbrugghe, 2011), aggregation of volumes differ whether they are estimated at world prices or at domestic prices. Hence, the shadow price of bilateral trade, $PDEM_{i,s,r,t}$, is evaluated as follow:

$$PDEM_{i,s,r,t} = PCIF_{i,s,r,t}\,(1 + DD_{i,s,r,t}) \tag{51}$$

which leads to the definition of the aggregator $TRADE_{i,s,r,t}$:

$$DEMA_{i,s,r,t}\, PDEMA_{i,s,r,t} = PDEM_{i,s,r,t}\, TRADE_{i,s,r,t} \tag{52}$$

Figure 2 : Demand by geographic origin[5]



2.4.5 Demand for transportation services

The volume of transportation $Tr_{i,s,r,t}$ required to move commodity i imported by region r from region s is a fixed proportion $MUO_{i,s,r}$ of total imports $TRADE_{i,s,r,t}$.

$$Tr_{i,s,r,t} = MUO_{i,s,r}\ TRADE_{i,s,r,t} \tag{53}$$

Transportation demand per mode, $TrMode_{Transport,i,s,r,t}$, is then determined as being a fixed share $a^{Tr}_{Transport,i,s,r}$ of total transportation demand. Implicitly, thus, total demand for transportation is a Cobb-Douglas type of function. Hence, the exact price formulation for the aggregated price of transportation, $PTr_{i,s,r,t}$, is the dual form of a Cobb-Douglas.

$$PTrMode_{Transport,t}\ TrMode_{Transport,i,s,r,t} = a^{Tr}_{Transport,i,s,r,t}\ Tr_{i,s,r,t}\ PTr_{i,s,r,t} \tag{54}$$

$$PTr_{i,s,r,t} = \prod_{Transport} PTrMode_{Transport,t}^{a^{Tr}_{Transport,i,s,r}} \tag{55}$$

with

$PTrMode_{Transport,t}$	Price of transport per mode
$PTr_{i,r,s,t}$	Price of transportation by commodity and partners

[5] The acronyms for the volume followed by its corresponding price appear in brackets.

2.5 Supply and market clearing

2.5.1 Transportation market

The world supply of transportation services per mode, $WorldTr_{Transport,t}$, follows a Cobb-Douglas specification. It follows that the supply from each region, $TrSupply_{Transport,r,t}$, is a constant share of the world value of transportation.

$$WorldTr_{Transport,t} = c^{T}_{Transport} \prod_{r} TrSupply_{Transport,r,t}^{a^{TrSupply}_{Transport,r}} \tag{56}$$

$$PY_{Transport,r,t}\left(1 + taxP_{Transport,r,t}\right) TrSupply_{Transport.r.t} = a^{TrSupply}_{Transport,r}\, PTrMode_{Transport,t}\, WorldTr_{Transport,t} \tag{57}$$

with

$c^{T}_{Transport}$	Scale coefficient
$a^{TrSupply}_{Transport,r}$	Share of each region in the world transport production

Market for transportation clears, since demand of transportation is equal to supply. Equilibrium on the transportation market determines the world prices of transportation per mode, $PTrMode_{Transport,t}$.

$$WorldTr_{Transport,t} = \sum_{i,r,s} TrMode_{Transport,i,r,s,r} \tag{58}$$

2.5.2 Commodity market

In each region, supply of each commodity is equal to demand. Market clearing determines the price of each commodity, $PY_{i,r,t}$.

$$Y_{i,r,t} = D_{i,r,t} + \sum_{s} TRADE_{i,r,s,t} + TrSupply_{i,r,t} \tag{59}$$

2.5.3 Factors of production market

2.5.3.1 Labor market

Total supply of skilled workers, $\overline{H}_{r,t}$, is fixed and grows exogenously. Skilled workers are assumed to be perfectly mobile across sectors and there is no unemployment. Hence, the equilibrium between supply and demand determines the wage rate.

$$\overline{H}_{r,t} = \sum_{j} H_{j,r,t} \tag{60}$$

The same assumption is followed regarding total supply of unskilled workers ($\bar{L}_{r,t}$): it is fixed and grows at an exogenous rate. However, it is assumed that unskilled workers cannot move freely between rural and urban areas. A CET (Constant elasticity of transformation) is used to characterize the regional supply of unskilled workers. Unskilled workers maximize their income subject to the CET aggregator, which leads to the following supply function:

$$Lt_{Ltype,r,t} = b^{Lt}_{Ltype,r} \, \bar{L}_{r,t} \left(\frac{WLt_{Ltype,r,t}}{\overline{WL}_{r,t}}\right)^{\sigma^L} \qquad (61)$$

with

$Lt_{Ltype,r,t}$	Labor supply on the Ltype market
$b^{Lt}_{Ltype,r}$	Labor scale coefficient
$\overline{WL}_{r,t}$	Aggregated wage for unskilled workers
σ^L	Labor elasticity

It follows that the aggregated wage for unskilled workers $\overline{WL}_{r,t}$ is a weighted sum of the wages received on each market:

$$\overline{WL}_{r,t} \, \bar{L}_{r,t} = \sum_{Ltype} WLt_{Ltype,r,t} \, Lt_{Ltype,r,t} \qquad (62)$$

which is determined by the equilibrium between supply and demand.

$$Lt_{Ltype,r,t} = \sum_{j} L_{j,r,t} \qquad (63)$$

2.5.3.2 Land market

Land mobility across sectors is assumed to be imperfect. Land supply, $\overline{TE}_{r,t}$, behaves as an isoelastic function of the real return to land (Lee and Mensbrugghe, 2001)). This implies that the greater the real overall return to land, the greater will be the overall supply of land.

$$\overline{TE}_{r,t} = \overline{TE}^{O}_{r} \left(\frac{\overline{WTE}_{r,t}}{P_{r,t}}\right)^{\sigma^{TE}_{r}} \qquad (64)$$

with

$\overline{TE}^{O}_{r}$	Benchmark value of total land supply
$\overline{WTE}_{r,t}$	Aggregated price for land
σ^{TE}_{r}	Total land supply elasticity

To represent the imperfect mobility of land, supply to each activity, $TE_{j,r,t}$, is determined following a CET aggregation. Land owners maximize their income subject to the CET aggregator, which leads to the following first order condition:

$$TE_{j,r,t} = b_{j,r}^{TE}\ \overline{TE}_{r,t} \left(\frac{WTE_{j,r,t}}{\overline{WTE}_{r,t}}\right)^{\sigma^{TE}} \tag{65}$$

with

$b_{j,r}^{TE}$	Land scale coefficient
σ^{TE}	Land elasticity

It follows that the aggregated price of land is weighted sum of the price received in each activity.

$$\overline{WTE}_{r,t}\ \overline{TE}_{r,t} = \sum_{j} WTE_{j,r,t}\ TE_{j,r,t} \tag{66}$$

2.5.3.3 Capital market

At each period, the capital stock invested by region s in activity j in region r, $K_{j,s,r,t}$, is given by the depreciated stock of capital inherited from the preceding period plus new investment $INV_{j,s,r,t}$

$$K_{j,s,r,t} = K_{j,s,r,t-1}\,(1-\delta_r) + INV_{j,s,r,t} \tag{67}$$

with

δ_r	Depreciation rate

Where the investment per activity and region of destination depends on the rate of return to capital, the aggregated price of new capital and capital stock[6].

$$INV_{j,s,r,t} = B_{s,t}\ a_{j,s,r}\ KTOT_{j,r,t}\ e^{\alpha\left(\frac{WK_{j,r,t}}{PINVTOT_{r,t}}\right)} \tag{68}$$

with

$B_{s,t}$	Scale coefficient for investment
$a_{j,s,r}$	Investment scale coefficient
α	Elasticity of investment to return on capital

Total investment made in region r, $INVTOT_{r,t}$, is simply the sum of investment made in each sector of each region:

$$INVTOT_{r,t} = \sum_{j,s} INV_{j,s,r,t} \tag{69}$$

[6] For a complete discussion on the investment behaviour, see Decreux and Valin (2007).

In each sector, total supply of capital equals demand, which determines the rate of return to capital specific to this sector ($WK_{i,r,t}$).

$$KTOT_{i,r,t} = \sum_{s} K_{i,s,r,t} \tag{70}$$

2.6 Macroeconomic constraints

In each region, total investment must be equal to total savings:

$$SAVH_{r,t} + SAVG_{r,t} - CAB_{r,t} = \sum_{i,s} PINVTOT_{s,t}\, INV_{i,r,s,t} \tag{71}$$

Where $CAB_{r,t}$ represents the current account balance, which is a constant share $SOLD_{r,t}$ of world GDP, $PIBMVAL_t$.

$$CAB_{r,t} = PIBMVAL_t\, SOLD_{r,t} \tag{72}$$

World GDP is the simply the sum of regional GDPs, $GDPMR_{r,t}$:

$$PIBMVAL_t = \sum_{r} GDPMP_{r,t} \tag{73}$$

Consistent with the system of national accounting, each region's GDP at market prices is given by the sum of payments to factors of production and of indirect taxes.

$$GDPMP_{r,t} = \sum_{j} PVA_{j,r,t}\, VA_{j,r,t} + \sum_{i} \{RECPROD_{i,r,t}\, RECEXP_{i,r,t}\, RECDD_{i,r,t}\, RECCONS_{i,r,t}\} \tag{74}$$

Finally, real GDP, $GDPVOL_{r,t}$, is computed by dividing GDP at market prices by a consumer price index:

$$GDPVOL_{r,t} = \frac{GDPMP_{i,r}}{\prod_i PC_{i,r,t}\, pondC_{i,r}} \tag{75}$$

2.7 Economic Closures

In MIRAGRODEP, every economic agent balances income and expenditures: income of households equals to spending of households (consumption, savings and transfers), firms' spending (including payment to capital) equals firms' revenue. At a global level, savings must be equal to investment. At the country level, a gap between the two variables can occur due to international capital movements. Nevertheless, constraints on current account surplus or deficits are also considered, leading to real exchange rate adjustments (determining relative international prices among economies). Furthermore, supply equals demand for all commodities and factors in the economy.

3. Summary of Model Structure

Table 1: Equations of MIRAGRODEP

Production		
First level: Leontief		GAMS
1.	$Y_{j,r,t} = a_{j,r}^{VA} VA_{j,r,t}$	EQ_VA
2.	$Y_{j,r,t} = a_{j,r}^{CNTER} CNTER_{j,r,t}$	EQ_CNTER
3.	$PY_{j,r,t}\, Y_{j,r,t} = PVA_{j,r,t}\, VA_{j,r,t} + PCNTER_{j,r,t}\, CNTER_{j,r,t}$	EQ_Y
Second level – Value added: CES		GAMS
4.	$L_{j,r,t} = a_{j,r}^{L}\, VA_{j,r,t}\, PGF_{r,t}{}^{\sigma_j^{VA}-1} \left(\frac{PVA_{j,r,t}}{PL_{j,r,t}}\right)^{\sigma_j^{VA}}$	EQ_CES_L
5.	$TE_{j,r,t} = a_{j,r}^{TE} \cdot VA_{j,r,t} \cdot PGF_{r,t}{}^{\sigma_j^{VA}-1} \cdot \left(\frac{PVA_{j,r,t}}{PTE_{j,r,t}}\right)^{\sigma_j^{VA}}$	EQ_CES_TE
6.	$RN_{j,r,t} = a_{j,r}^{RN} \cdot VA_{j,r,t} \cdot PGF_{r,t}{}^{\sigma_j^{VA}-1} \left(\frac{PVA_{j,r,t}}{PRN_{j,r,t}}\right)^{\sigma_j^{VA}}$	EQ_CES_RN
7.	$Q_{j,r,t} = a_{j,r}^{Q} \cdot VA_{j,r,t} \cdot PGF_{r,t}{}^{\sigma_j^{VA}-1} \cdot \left(\frac{PVA_{j,r,t}}{PQ_{j,r,t}}\right)^{\sigma_j^{VA}}$	EQ_CES_Q
8.	$PVA_{j,r,t}\, VA_{j,r,t} = PL_{j,r,t}\, L_{j,r,t} + PTE_{j,r,t}\, TE_{j,r,t} + PRN_{j,r,t}\, RN_{j,r,t} + PQ_{j,r,t}\, Q_{j,r,t}$	EQ_CES_PVA
9.	$PL_{j,r,t} = WLt_{Ltype,r,t} \left(1 + taxf_{UnSkLab,j,r,t}^{VAL}\right) + PIndC_{r,t}\, taxf_{UnSkLab,j,r,t}^{VOL}$	EQ_PL
10.	$PTE_{j,r,t} = WTE_{j,r,t} \left(1 + taxf_{Land,j,r,t}^{VAL}\right) + PIndC_{r,t}\, taxf_{Land,j,r,t}^{VOL}$	EQ_PTE
11.	$PRN_{j,r,t} = WRN_{j,r,t} \left(1 + taxf_{NatlRes,j,r,t}^{VAL}\right) + PIndC_{r,t}\, taxf_{NatlRes,j,r,t}^{VOL}$	EQ_PRN
Third level – Capital-Skilled labor bundle: CES		GAMS
12.	$H_{j,r,t} = a_{j,r}^{H}\, Q_{j,r,t} \left(\frac{PQ_{j,r,t}}{PH_{j,r,t}}\right)^{\sigma_j^{CAP}}$	EQ_H
13.	$KTOT_{j,r,t} = a_{j,r}^{K}\, Q_{j,r,t} \left(\frac{PQ_{j,r,t}}{PK_{j,r,t}}\right)^{\sigma_j^{CAP}}$	EQ_KTOT

14.	$PQ_{j,r,t}\ Q_{j,r,t} = PK_{j,r,t}\ KTOT_{j,r,t} + PH_{j,r,t}\ H_{j,r,t}$	EQ_PQ
15.	$PH_{j,r,t} = WH_{r,t}\left(1 + taxf^{VAL}_{SkLab,j,r,t}\right) + PIndC_{r,t}\ taxf^{VOL}_{SkLab,j,r,t}$	EQ_PH
16.	$PK_{j,r,t} = WK_{j,r,t}\left(1 + taxf^{VAL}_{Capital,j,r,t}\right) + PIndC_{r,t}\ taxf^{VOL}_{Capital,j,r,t}$	EQ_PK
Second level – Intermediate consumption: CES		GAMS
17.	$IC_{i,j,r,t} = a^{IC}_{i,j,r} CNTER_{j,r,t}\left(\frac{PCNTER_{j,r,t}}{PIC_{i,j,r,t}}\right)^{\sigma^{IC}}$	EQ_IC
18.	$PCNTER_{j,r,t}\ CNTER_{j,r,t} = \sum_i PIC_{i,j,r,t}\ IC_{i,j,r,t}$	EQ_PCNTER
19.	$PIC_{i,j,r,t} = PDEMTOT_{i,r,t}\ (1 + taxicc_{i,j,r,t})$	EQ_PIC
Income and Savings		
Households		GAMS
20.	$REVH_{r,t} = \sum_j \left\{ WRN_{j,r,t}\ RN_{j,r,t} + WTE_{j,r,t}\ TE_{j,r,t} + WH_{r,t}\ H_{j,r,t} + \sum_{Ltype} WLt_{Ltype,r,t}\ L_{j,r,t} + \sum_s WK_{j,s,t}\ K_{j,r,s,t} \right\} + Pop^{ag}_{totpop,r,t}\ TRH_{r,t}\ PIndC_{r,t}$	EQ_REVH
21.	$SAVH_{r,t} = epa_r\ (REVH_{r,t} - RECDIR_{r,t})$	EQ_SAVH
22.	$BUDH_{r,t} = REVH_{r,t} - SAVH_{r,t} - RECDIR_{r,t}$	EQ_BUDH
Government		GAMS
23.	$REVG_{r,t} = \sum_i \{RECPROD_{i,r,t} + RECFAC_{i,r,t} + RECEXP_{i,r,t} + RECDD_{i,r,t} + RECCONS_{i,r,t}\} + RECDIR_{r,t}$	EQ_REVG
24.	$RECPROD_{i,r,t} = taxP_{i,r,t}\ PY_{i,r,t}\ Y_{i,r,t}$	EQ_RECPROD

25.	$RECFAC_{j,r,t} = PIndC_{r,t}\Big(taxf^{VOL}_{Land,j,r,t}\,TE_{j,r,t} + taxf^{VOL}_{NatlRes,j,r,t}\,RN_{j,r,t} + taxf^{VOL}_{SkLab,j,r,t}\,H_{j,r,t} + taxf^{VOL}_{UnSkLab,j,r,t}\,L_{j,r,t} + taxf^{VOL}_{Capital,j,r,t}\,KTOT_{j,r,t}\Big) + taxf^{VAL}_{Land,j,r,t}\,WTE_{j,r,t}\,TE_{j,r,t} + taxf^{VAL}_{NatlRes,j,r,t}\,WRN_{j,r,t}\,RN_{j,r,t} + taxf^{VAL}_{SkLab,j,r,t}\,WH_{r,t}\,H_{j,r,t} + taxf^{VAL}_{UnSkLab,j,r,t}\sum_{Ltype} WLt_{Ltype,r,t}\,L_{j,r,t} + taxf^{VAL}_{Capital,j,r,t}\,WK_{j,r,t}\,KTOT_{j,r,t}$	EQ_RECFAC
26.	$RECEXP_{i,r,t} = PY_{i,r,t}(1 + taxP_{i,r,t})\sum_{s}(taxEXP_{i,r,s,t} + taxAMF_{i,r,s,t})TRADE_{i,r,s,t}$	EQ_RECEXP
27.	$RECDD_{i,r,t} = \sum_{s} DD_{i,s,r,t}\,PCIF_{i,s,r,t}\,TRADE_{i,s,r,t}$	EQ_RECDD
28.	$RECCONS_{i,r,t} = PDEMTOT_{i,r,t}\Big\{taxcc_{i,r,t}\,CH_{i,r,t} + taxgc_{i,r,t}\,CG_{i,r,t} + taxkgc_{i,r,t}\,KG_{i,r,t} + \sum_{j} taxicc_{i,j,r,t}\,IC_{i,j,r,t}\Big\}$	EQ_RECCONS
29.	$RECDIR_{r,t} = taxdir_{r,t}\,REVH_{r,t}$	EQ_RECDIR
30.	$SAVG_{r,t} = PUBSOLD_{r}\,GDPMP_{r,t}$	EQ_SAVG
31.	$BUDG_{r,t} = REVG_{r,t} - SAVG_{r,t} - Pop^{ag}_{totpop,r,t}\,TRH_{r,t}\,PIndC_{r,t}$	EQ_BUDG
Demand		
32.	$DEMTOT_{i,r,t} = CH_{i,r,t} + CG_{i,r,t} + \sum_{j} IC_{i,j,r,t} + KG_{i,r,t}$	EQ_DEMTOT
Private demand		GAMS
33.	$CH_{i,r,t} = Pop^{ag}_{totpop,r,t}\left(cmin_{i,r} + a^{C}_{i,r}\,AUX_{r,t}\left(\frac{P_{r,t}}{PC_{i,r,t}}\right)^{\sigma^{C}_{r}}\right)$	EQ_CH
34.	$BUDH_{r,t} = \sum_{i} PC_{i,r,t}\,CH_{i,r,t}$	EQ_AUX
35.	$P_{r,t}\,AUX_{r,t} = \sum_{i} PC_{i,r,t}\left(\frac{CH_{i,r,t}}{Pop^{ag}_{totpop,r,t}} - cmin_{i,r}\right)$	EQ_P
36.	$PC_{i,r,t} = PDEMTOT_{i,r,t}\,(1 + taxcc_{i,r,t})$	EQ_PC

37.	$PindC_{r,t} = \sqrt{\left[\frac{\sum_i PC_{i,r,t}\, CH^O_{i,r}}{\sum_i PC^O_{i,r}\, CH^O_{i,r}}\right]\left[\frac{\sum_i PC_{i,r,t}\, CH_{i,r,t}}{\sum_i PC^O_{i,r}\, CH_{i,r,t}}\right]}$	EQ_PI
Public demand		GAMS
38.	$PCG_{i,r,t} CG_{i,r,t} = \alpha^G_{i,r}\, BUDG_{r,t}$	EQ_CG
39.	$PCG_{i,r,t} = PDEMTOT_{i,r,t}\left(1 + taxgc_{i,r,t}\right)$	EQ_PCG
Demand for investment purposes		GAMS
40.	$KG_{i,r,t} = a^{KG}_{i,r}\, INVTOT_{r,t}\left(\frac{PINVTOT_{r,t}}{PKG_{i,r,t}}\right)^{\sigma KG}$	EQ_KG
41.	$PINVTOT_{r,t}\, INVTOT_{r,t} = \sum_i PKG_{i,r,t}\, KG_{i,r,t}$	EQ_PINVTOT
42.	$PKG_{i,r,t} = PDEMTOT_{i,r,t}\,(1 + taxkgc_{i,r,t})$	EQ_PKG
Demand by geographic origin		GAMS
43.	$D_{i,r,t} = a^D_{i,r}\, DEMTOT_{i,r,t}\left(\frac{PDEMTOT_{i,r,t}}{PD_{i,r,t}}\right)^{\sigma^{ARM}_i}$	EQ_D
44.	$M_{i,r,t} = a^M_{i,r}\, DEMTOT_{i,r,t}\left(\frac{PDEMTOT_{i,r,t}}{PM_{i,r,t}}\right)^{\sigma^{ARM}_i}$	EQ_M
45.	$PDEMTOT_{i,r,t}\, DEMTOT_{i,r,t} = PD_{i,r,t}\, D_{i,r,t} + PM_{i,r,t}\, M_{i,r,t}$	EQ_PDEMTOT
46.	$PD_{i,r,t} = PY_{i,r,t}\,(1 + taxP_{i,r,t})$	EQ_PD
47.	$DEMA_{i,s,r,t} = a^{IMP}_{i,s,r}\, M_{i,r,t}\left(\frac{PM_{i,r,t}}{PDEMA_{i,s,r,t}}\right)^{\sigma^{IMP}_i}$	EQ_DEMA
48.	$PM_{i,r,t}\, M_{i,r,t} = \sum_s PDEMA_{i,s,r,t}\, DEMA_{i,s,r,t}$	EQ_PM
49.	$PDEMA_{i,s,r,t} = PCIF_{i,s,r,t}\left(1 + DD^A_{i,s,r,t}\right)$	EQ_PDEMA
50.	$PCIF_{i,s,r,t} = PY_{i,s,t}\,(1 + taxEXP_{i,s,r,t} + taxAMF_{i,s,r,t})\,(1 + taxP_{i,s,t}) + MUO_{i,s,r}\, PTr_{i,s,r,t}$	EQ_PCIF
51.	$PDEM_{i,s,r,t} = PCIF_{i,s,r,t}\,(1 + DD_{i,s,r,t})$	EQ_PDEM
52.	$DEMA_{i,s,r,t}\, PDEMA_{i,s,r,t} = PDEM_{i,s,r,t}\, TRADE_{i,s,r,t}$	EQ_TRADE

Demand for transportation services		GAMS
53.	$Tr_{i,s,r,t} = MUO_{i,s,r}\ TRADE_{i,s,r,t}$	EQ_Tr
54.	$PTrMode_{Transport,t}\ TrMode_{Transport,i,s,r,t} = a^{Tr}_{Transport,i,s,r,t}\ Tr_{i,s,r,t}\ PTr_{i,s,r,t}$	EQ_TrMode
55.	$PTr_{i,s,r,t} = \prod_{Transport} PTrMode_{Transport,t}^{a^{Tr}_{Transport,i,s,r}}$	EQ_PTr
Supply and market clearing		
Transportation market		GAMS
56.	$WorldTr_{Transport,t} = c^{T}_{Transport} \prod_{r} TrSupply_{Transport,r,t}^{a^{TrSupply}_{Transport,r}}$	EQ_WorldTr
57.	$PY_{Transport,r,t}\left(1 + taxP_{Transport,r,t}\right) TrSupply_{Transport.r.t} = a^{TrSupply}_{Transport,r}\ PTrMode_{Transport,t}\ WorldTr_{Transport,t}$	EQ_TrSupply
58.	$WorldTr_{Transport,t} = \sum_{i,r,s} TrMode_{Transport,i,r,s,r}$	EQ_PTrMode
Commodity market		GAMS
59.	$Y_{i,r,t} = D_{i,r,t} + \sum_{s} TRADE_{i,r,s,t} + TrSupply_{i,r,t}$	EQ_PY
Factors of production market		
Labor market		GAMS
60.	$\overline{H}_{r,t} = \sum_{j} H_{j,r,t}$	EQ_WH
61.	$Lt_{Ltype,r,t} = b^{Lt}_{Ltype,r}\ \overline{L}_{r,t} \left(\frac{WLt_{Ltype,r,t}}{\overline{WL}_{r,t}}\right)^{\sigma^{L}}$	EQ_CET_Lt
62.	$\overline{\mathrm{WL}}_{\mathrm{r,t}}\ \overline{\mathrm{L}}_{\mathrm{r,t}} = \sum_{\mathrm{Ltype}} \mathrm{WLt}_{\mathrm{Ltype,r,t}}\ \mathrm{Lt}_{\mathrm{Ltype,r,t}}$	EQ_CET_WLbar
63.	$\mathrm{Lt}_{\mathrm{Ltype,r,t}} = \sum_{\mathrm{j}} \mathrm{L}_{\mathrm{j,r,t}}$	EQ_WLt
Land market		GAMS
64.	$\overline{\mathrm{TE}}_{\mathrm{r,t}} = \overline{\mathrm{TE}}^{\mathrm{O}}_{\mathrm{r}} \left(\frac{\mathrm{WTE}_{\mathrm{r,t}}}{\mathrm{P}_{\mathrm{r,t}}}\right)^{\sigma^{\mathrm{TE}}_{\mathrm{r}}}$	EQ_TEbar

65.	$\mathrm{TE}_{j,r,t} = b^{TE}_{j,r}\,\overline{\mathrm{TE}}_{r,t}\left(\frac{\mathrm{WTE}_{j,r,t}}{\overline{\mathrm{WTE}}_{r,t}}\right)^{\sigma^{TE}}$	EQ_CET_WTE
66.	$\overline{\mathrm{WTE}}_{r,t}\,\overline{\mathrm{TE}}_{r,t} = \sum_{j} \mathrm{WTE}_{j,r,t}\,\mathrm{TE}_{j,r,t}$	EQ_CET_WTEbar
Capital market		GAMS
67.	$K_{j,s,r,t} = K_{j,s,r,t-1}\,(1-\delta_r) + INV_{j,s,r,t}$	EQ_K
68.	$\mathrm{INV}_{j,s,r,t} = B_{s,t}\,a_{j,s,r}\,\mathrm{KTOT}_{j,r,t}\,e^{\alpha\left(\frac{\mathrm{WK}_{j,r,t}}{\mathrm{PINVTOT}_{r,t}}\right)}$	EQ_INV
69.	$\mathrm{INVTOT}_{r,t} = \sum_{j,s} \mathrm{INV}_{j,s,r,t}$	EQ_INVTOT
70.	$\mathrm{KTOT}_{i,r,t} = \sum_{s} \mathrm{K}_{i,s,r,t}$	EQ_WK
Macroeconomic constraints		
71.	$\mathrm{SAVH}_{r,t} + \mathrm{SAVG}_{r,t} - \mathrm{CAB}_{r,t} = \sum_{i,s} \mathrm{PINVTOT}_{s,t}\,\mathrm{INV}_{i,r,s,t}$	EQ_B
72.	$CAB_{r,t} = PIBMVAL_t\,SOLD_{r,t}$	EQ_CAB
73.	$\mathrm{PIBMVAL}_t = \sum_{r} \mathrm{GDPMP}_{r,t}$	EQ_PIBMVAL
74.	$\mathrm{GDPMP}_{r,t} = \sum_{j} \mathrm{PVA}_{j,r,t}\,\mathrm{VA}_{j,r,t} + \sum_{i}\left\{\mathrm{RECPROD}_{i,r,t}\ \mathrm{RECEXP}_{i,r,t}\ \mathrm{RECDD}_{i,r,t}\ \mathrm{RECCONS}_{i,r,t}\right\}$	EQ_GDP
75.	$\mathrm{GDPVOL}_{r,t} = \frac{\mathrm{GDPMP}_{i,r}}{\prod_i \mathrm{PC}_{i,r,t}\,\mathrm{pondC}_{i,r}}$	EQ_PGF

Table 2: Variables of MIRAGRODEP

Variable	Definition	GAMS
$AUX_{r,t}$	Utility	`AUX(r,Temps,simul)`
$B_{r,t}$	Investment scale coefficient	`B(r,Temps,simul)`
$BUDG_{r,t}$	Budget allocated to public consumption	`BUDG(r,Temps,simul)`
$BUDH_{r,t}$	Budget allocated to private consumption	`BUDH(r,Temps,simul)`
$CAB_{r,t}$	Current account balance	`CAB(r,temps,simul)`
$CGi_{,r,t}$	Public consumption of commodity *i*	`CG(i,r,Temps,simul)`
$CH_{i,r,t}$	Consumption of commodity *i* by households	`CH(i,r,Temps,simul)`
$CNTER_{j,r,t}$	Aggregate intermediate consumption by sector *j*	`CNTER(j,r,Temps,simul)`
$D_{i,r,t}$	Demand for domestic commodity *i*	`D(i,r,Temps,simul)`
$DEMA_{i,r,s,t}$	Bilateral trade from *r* to *s* (volume)	`DEMA(i,r,s,Temps,simul)`
$DEMTOT_{i,r,t}$	Total demand for composite commodity *i*	`DEMTOT(i,r,Temps,simul)`
$GDPMP_{r,t}$	Gross domestic product at market prices (nominal)	`GDPMP(r,Temps,simul)`
$GDPVOL_{r,t}$	Gross domestic product at market prices (real)	`GDPVOL(r,Temps,simul)`
$H_{j,r,t}$	Demand for skilled labor by sector	`H(j,r,Temps,simul)`
$\bar{H}_{r,t}$	Total skilled labor supply	`Hbar(r,Temps,simul)`
$IC_{i,j,r,t}$	Intermediate consumption of good *i* by sector *j*	`IC(i,j,r,Temps,simul)`
$INV_{j,s,r,t}$	Investment made by *s* in sector *j* of region *r*	`INV(j,s,r,Temps,simul)`
$INVTOT_{r,t}$	Total investment in region *r*	`INVTOT(r,Temps,simul)`
$K_{j,s,r,t}$	Capital stock invested by *s* in *r*	`K(j,s,r,Temps,simul)`
$KG_{i,r,t}$	Demand of good *i* for investment purposes	`KG(i,r,Temps,simul)`
$KTOT_{j,r,t}$	Capital stock available in sector *j*	`KTOT(j,r,Temps,simul)`
$L_{j,r,t}$	Demand for unskilled labor by sector *j*	`L(j,r,Temps,simul)`
$\bar{L}_{r,t}$	Total supply of unskilled labor	`Lbar(r,Temps,simul)`
$Lt_{Ltype,r,t}$	Supply of unskilled labor per type	`Lt(Ltype,r,Temps,simul)`
$M_{i,r,t}$	Aggregate imports by region *r*	`M(i,r,Temps,simul)`
$P_{r,t}$	Price of utility	`P(r,Temps,simul)`
$PC_{i,r,t}$	Price of final private consumption	`PC(i,r,Temps,simul)`

$PCG_{i,r,t}$	Price of final public consumption	`PCG(i,r,Temps,simul)`
$PCIF_{i,r,s,t}$	CIF price	`PCIF(i,r,s,Temps,simul)`
$PCNTER_{j,r,t}$	Price of aggregate intermediate consumption by sector *j*	`PCNTER(j,r,Temps,simul)`
$PD_{i,r,t}$	Price of for domestic good *i* (including taxes)	`PD(i,r,Temps,simul)`
$PDEM_{i,r,s,t}$	Price of bilateral trade from *r* to *s*	`PDEM(i,r,s,Temps,simul)`
$PDEMA_{i,r,s,t}$	Price of bilateral trade from *r* to *s*	`PDEMA(i,r,s,Temps,simul)`
$PDEMTOT_{i,r,t}$	Price of composite commodity *i*	`PDEMTOT(i,r,Temps,simul)`
$PGF_{r,t}$	Total factor productivity	`PGF(r,Temps,simul)`
$PH_{j,r,t}$	Price of skilled labor (including taxes)	`PH(j,r,Temps,simul)`
$PIBMVAL_{t}$	World gross domestic product (value)	`PIBMVAL(Temps,simul)`
$PIC_{i,j,r,t}$	Price of intermediate consumption good *i* for sector *j* (including taxes)	`PIC(i,j,r,Temps,simul)`
$PIndC_{r,t}$	Consumer price index	`PIndC(r,Temps,simul)`
$PINVTOT_{r,t}$	Aggregate price of investment in region *r*	`PINVTOT(r,Temps,simul)`
$PK_{j,r,t}$	Price of capital (including taxes)	`PK(j,r,Temps,simul)`
$PKG_{i,r,t}$	Price of capital good consumption of good i (including taxes)	`PKG(i,r,Temps,simul)`
$PL_{j,r,t}$	Price of unskilled labor (including taxes)	`PL(j,r,Temps,simul)`
$PM_{i,r,t}$	Price of aggregate imports	`PM(i,r,Temps,simul)`
$PQ_{j,r,t}$	Price of capital - skilled labor aggregate	`PQ(j,r,Temps,simul)`
$PRN_{j,r,t}$	Price of natural resources (including taxes)	`PRN(j,r,Temps,simul)`
$PTE_{j,r,t}$	Price of land (including taxes)	`PTE(j,r,Temps,simul)`
$PTr_{i,r,s,t}$	Price of aggregate transport by export	`PTr(i,r,s,Temps,simul)`
$PTrMode_{j,t}$	World price of transport per mode	`PTrMode(j,Temps,simul)`
$PVA_{j,r,t}$	Price of value added	`PVA(j,r,Temps,simul)`
$PY_{j,r,t}$	Output price	`PY(j,r,Temps,simul)`
$Q_{j,r,t}$	Capital - skilled labor aggregate	`Q(j,r,Temps,simul)`
$RECCONS_{i,r,t}$	Consumption tax receipts	`RECCONS(i,r,Temps,simul)`
$RECDD_{i,r,t}$	Tariff revenues	`RECDD(i,r,Temps,simul)`
$RECDIR_{r,t}$	Tax receipts from direct taxation	`RECDIR(r,Temps,simul)`
$RECEXP_{i,r,t}$	Export tax receipts	`RECEXP(i,r,Temps,simul)`
$RECFAC_{j,r,t}$	Receipts from taxes on factors of production	`RECFAC(j,r,Temps,simul)`

$RECPROD_{i,r,t}$	Production tax receipts	RECPROD(i,r,Temps,simul)
$REVG_{r,t}$	Government total income	REVG(r,Temps,simul)
$REVH_{r,t}$	Households income	REVH(r,Temps,simul)
$RN_{j,r,t}$	Demand for natural resources by sector	RN(j,r,Temps,simul)
$SAVG_{r,t}$	Government savings	SAVG(r,Temps,simul)
$SAVH_{r,t}$	Households savings	SAVH(r,Temps,simul)
$SOLD_{r,t}$	Initial share of current account balance in world GDP	SOLD(r,Temps,simul)
$TE_{j,r,t}$	Land used in sector *j*	TE(j,r,Temps,simul)
$\overline{TE}_{r,t}$	Total land supply	TEbar(r,Temps,simul)
$Tr_{i,r,s,t}$	Transport demand by export	Tr(i,r,s,Temps,simul)
$TRADE_{i,r,s,t}$	Bilateral trade from *r* to *s* (volume)	TRADE(i,r,s,Temps,simul)
$TRH_{r,t}$	Public transfers to households	TRH(r,Temps,simul)
$TrMode_{j,i,r,s,t}$	Transport demand by export, per mode	TrMode(j,i,r,s,Temps,simul)
$TrSupply_{j,r,t}$	Supply of international transportation by region *r*	TrSupply(j,r,Temps,simul)
$VA_{j,r,t}$	Value added	VA(j,r,Temps,simul)
$WH_{r,t}$	Rate of return to skilled labor	WH(r,Temps,simul)
$WK_{i,r,t}$	Rate of return to capital	WK(i,r,Temps,simul)
$\overline{WL}_{r,t}$	Price of aggregate unskilled labor	WLbar(r,Temps,simul)
$WLt_{Ltype,r,t}$	Rate of return to unskilled labor	WLt(Ltype,r,Temps,simul)
$WorldTr_{j,t}$	World supply of international transportation	WorldTr(j,Temps,simul)
$WRN_{j,r,t}$	Rate of return to natural resources	WRN(j,r,Temps,simul)
$WTE_{j,r,t}$	Rate of return to land	WTE(j,r,Temps,simul)
$\overline{WTE}_{r,t}$	Aggregate price of land	WTEbar(r,Temps,simul)
$Y_{j,r,t}$	Total output of sector *j*	Y(j,r,Temps,simul)

4. References

Decreux, Y., & Valin, H. (2007). MIRAGE, Updated Version of the Model for Trade Policy Analysis Focus on Agriculture and Dynamics. CEPII Working Paper 2007- 15, October 2007, *CEPII*.

FAO (2013). FAOSTAT. Available Online at <http://faostat.fao.org/>

Femenia, F. (2012). Functional Forms Commonly Used in CGE Models. AGRODEP Technical Note 02. Washington, DC: International Food Policy Research Institute. Available Online at <http://www.agrodep.org/resource/functional-forms-commonly- used-cge-models>

Foure, J., A. Benassy-Quere and L. Fontagne (2012), The Great Shift: Macroeconomic Projections for the World Economy at the 2050 Horizon, CEPII Working paper 2012-03. Available Online at <http://www.cepii.fr/anglaisgraph/bdd/baseline.htm#sthash.LTHY86Pg.dpuf>

Laborde, D., Martin, W., & van der Mensbrugghe, D. (2011). Measuring the Impacts of Global Trade Reform with Optimal Aggregators of Distortions. Washington, D.C.

Lee, H., & Mensbrugghe, D. Van Der (2001). Interactions between Direct Investment and Trade in the Asia-Pacific Region. Presented at the 4th Annual Conference on Global Economic Analysis, Purdue University, USA. Purdue University, West Lafayette, IN: Global Trade Analysis Project (GTAP).

ILO (2013). International Labor Organization. Statistics and Databases. Available Online <http://www.ilo.org/global/statistics-and-databases/lang--en/index.htm>

IMF (2013). International Monetary Fund. World Economic Outlook Database. Available Online at <https://www.imf.org/external/pubs/ft/weo/2013/01/weodata/index.aspx>

Narayanan, B. G. and T. L. Walmsley, Editors (2008). *Global Trade, Assistance, and Production: The GTAP 7 Data Base*, Center for Global Trade Analysis, Purdue University.