AGRODEP Technical Note 0022

October 2021

Detecting Threshold Effects in Price Transmission

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Abstract

The analysis of price transmission plays a key role in understanding markets integration. This helps identify the nature of the relationship between geographically distant markets and cross-commodity price transmission, as well as the impact of liberalization policies and the identification of regions exposed to systemic shocks. This technical note contributes to the debate between symmetric and asymmetric price transmission and proposes to present the traditional and new approaches for detecting threshold effects in price transmission while focusing on their advantages and limitations. There is no one-size-fits-all method to detect threshold effects in price transmission. Experts need to select a combination of elements (context of study, the economy under consideration, data availability…) to justify the relevancy of their choice. Beyond the presentation of the methods for detecting thresholds in price transmission, we perform an application in the case of the rice market in Senegal. The results support the evidence of an asymmetric price transmission between world and domestic prices in the short-run and a symmetric transmission in the long-run.

Résumé

L'analyse de la transmission des prix joue un rôle clé dans la compréhension de l'intégration des marchés. Elle permet de mieux cerner la nature des relations entre les marchés géographiquement distants et la transmission des prix entre produits, d'analyser l'impact des politiques de libéralisation, ainsi que d'identifier les régions exposées aux chocs systémiques. Cette note technique se propose de donner une vue d’ensemble sur la littérature de la transmission symétrique et asymétrique des prix, ainsi que la présentation des méthodes traditionnelles et nouvelles pour la détection des effets d'asymétrie tout en mettant le focus sur leurs avantages et leurs limites. En principe, il n'existe pas de méthode unique pour détecter les effets asymétriques. L'analyste doit sélectionner une combinaison d'éléments (contexte de l'étude, économie considérée, disponibilité des données...) pour justifier la pertinence de son choix. Au-delà de la présentation des méthodes de détection des effets d'asymétrie, nous proposons une application au cas du marché du riz au Sénégal. Les résultats soutiennent la transmission asymétrique du prix du riz entre le marché international et le marché domestique à court terme et une transmission symétrique à long terme.

1. Introduction

The food crises of the mid 70s and of 2007-2008 affected the world with significant impacts on agricultural commodity prices. Securing access of these commodities, promoting markets integration, and providing price stability increases the food security of the main importing countries, particularly developing countries. Understanding the relationship between domestic and international or regional agricultural commodity prices requires understanding factors influencing supply and demand and assessing the degree of markets integration. For proponents of free trade, the more integrated the markets, the better the price stability and the well-being of economic agents as well as the absorption of systemic shocks[1] by domestic and regional markets (Ravallion, 1986, 1997; Sen 1981). On the other hand, for others, less optimistic, markets integration can lead to complex redistributive effects[2] (Newbery, Stiglitz, 1984; Bonjean and Combes, 2010). This shows that the analysis of market integration is a powerful framework to analyze the impact of liberalization policies, as well as the identification of regions exposed to systemic shocks. Indeed, the most widespread approach to analyzing market integration is the price transmission analysis.

Understanding the price transmission mechanism between markets geographically distant or potentially linked by other factors is important for policymakers to ensure price stability, the well-being of economic agents, as well as the absorption of systemic shocks. Indeed, low price transmission can lead to sub-optimal decisions for economic agents and create inefficiencies in policies implementation. The price transmission analysis is based explicitly, or at least implicitly, on the Enke-Samuelson-Takayama-Judge (ESTJ) spatial equilibrium model (Enke, 1951; Samuelson, 1952; Takayama and Judge, 1971). This model assumes the free movement of products and perfect information between geographically distinct markets. Subsequently, two major approaches have been developed to analyze this model.

The first, that of the law of one price (LOOP), is the analytical framework used the most to test markets integration (Richardson, 1978; Crouhy-Veyrac and al 1982; Ravallion 1986; Carter and Hamilton 1989; Goodwin and Schroeder, 1991; Sexton and al., 1991). According to Dornbusch (1987), the law of one price is consistent with the spatial integration of markets once transaction costs and real frictions are considered. The latter, in its relative version, stipulates that if there is trade of a product between two regions, the price of the importing region is equal to the price of the exporting region adjusted for transaction costs. However, information flows between markets and networks of traders can also allow the transmission of price signals between markets in the absence of trade flows (Jensen, 2007; Fackler and Tastan, 2008, Stephens and al., 2008; Ihle and al., 2010). The second, developed by McNew (1996), is based on an analysis of markets connectedness. This approach analyzes the dynamics of transmission of price shocks, especially the price

[1] Food crisis in Niger in 2005
[2] Market integration is profitable for producers of exportable products who benefit from better remuneration for their products, but producers of import substitute products lose.

adjustment process, while the model based on the law of the single price focuses on the price relationships between markets. The connectedness approach aims to measure the degree to which a price shock is transmitted from one region to another (McNew and Fackler, 1997). Moreover, due to market imperfections, the asymmetric prices transmission analyzes the mechanism by which upward and downward price shocks are different.

According to Peltzman (2000), the asymmetry is the rule rather than the exception. Several studies analyzing the price asymmetric transmission found that there is an asymmetric price transmission between international and domestic markets (Balke and Fomby, 1997; von Cramon-Taubadel, 1998; Abdulaï, 2000; Goodwin and Piggott, 2001; Badolo 2012). Increases in the international price are transmitted quicker to domestic prices than any decreases.

According to Gauthier et Zapata (2001) and Cramon-Taubadel et Meyer (2000), Meyer and Cramon-Taubadel (2004), the existing literature of asymmetric price transmission is far from being unified or conclusive, and is mostly method-driven, with little attention devoted to theoretical underpinnings and the plausible interpretation of results. Hence, there is scape for interesting theoretical and empirical work. However, Frey and Manera (2007) sum up the empirical findings as follows. In all its forms, asymmetric price transmission is likely to occur in a wide range of markets and econometric models.

The results of empirical research with respect to symmetrical or asymmetrical transmission vary greatly, depending on the sector tested, the country considered, the methodological approach and/or the frequency of data used in the analysis (Taubadel et al., 2006). Asymmetries in price transmission have been detected in some countries and sectors but not in others. Backus et al (2014) conclude that the local circumstances are determinant in the presence of asymmetric price transmission. However, the main factors developed to be the causes of asymmetric price transmission are market power of commercial intermediaries, transport costs, and government interventions.

Various studies on price transmission, particularly asymmetric price transmission, that develop large methodologies are not frequent, and those that integrate machine learning methods are not very common. The relevance of the type of tool used to detect the threshold in price transmission is necessary to ensure the consistency and the accuracy of asymmetric price transmission analysis.

This technical note presents the methods used to detect threshold effects in price transmission. The first section gives an overview of threshold effects in price transmission. The second section shows the traditional methods developed to detect threshold effects in price transmission. Finally, we present the new approaches developed using machine learning tools to detect threshold effects in price transmission.

2. Threshold effects in price transmission

2.1 Fundamental concepts

Prices play a key role in economic theory and need to be considered in any policy decision affecting both consumers and producers. According to Minot (2011), price transmission refers to the effect of prices in one market on prices of another market. To better understand the nature of price movements, economists have made efforts to analyze the nature, extent, direction and speed with which price movements are transmitted between spatially separated markets (international markets to domestic markets and regional markets or vice versa), and along the various stages of the agro-food chain (from farm to processing and retail levels or vice versa). The former transmission is known as "horizontal transmission" while the latter is referred to as "vertical transmission.".

2.1.1 Horizontal and vertical price transmission

All definitions for horizontal and vertical price transmission are unified and conclusive. Vertical price transmission refers to price linkages along a given supply chain, and horizontal price transmission means the linkage occurring among different markets at the same position in the supply chain (Listorti and Esposti, 2012). The notion of horizontal price transmission usually refers to price linkages across markets places (spatial price transmission), but it can also refer the transmission across different agricultural commodities (cross-commodity price transmission) (Esposti and Listorti, 2011), from non-agricultural to agricultural commodity (from energy/oil prices to agricultural prices), and across different purchase contracts for the same products (typically, from futures to spot markets and vice versa; Baldi et al., 2011).

In case of two vertical integrated markets, a price change at one market is diffused on the price of the other market. Referring on the derived demand theory, at least one input factor will be affected upstream or downstream. However, the background of spatial price transmission relies on the spatial arbitrage and results from the Law Of One Price (LOOP), while the cross price transmission refers to the substitutability between and complementary relations among commodities (Singh et al, 2015).

2.1.2 Symmetric and Asymmetric price transmission

The symmetry assumption of price transmission relies on the fact that, between two integrated markets, the mechanism of price transmission shouldn't be different from price increasing scenarios to price decreasing scenarios. However, if the speed of adjustment differs from price increases to price decreases, there is a phenomenon of asymmetric price transmission. For instance, when both world price increases and decreases are transmitted in the same way to domestic prices, then symmetric price transmission occurs. But asymmetric price transmission is introduced when world price increases and decreases are transmitted differently to domestic price.

Short-run and long-run relations analysis is equally important to consider in both symmetric and asymmetric analysis. *In general, an SR analysis is appropriate to compare the intensity of output price variations to positive or negative changes in input prices, whereas a LR is applicable if the empirical investigation concentrates on the computation of reaction times, length of fluctuations, as well as speeds of adjustment toward an equilibrium level (Frey and Manera, 2005)*

2.2 Reasons behind threshold effects

So far, the assumption is that the price in one market influences the price of the other market in a symmetric fashion (Goshray, 2011). However, the reasons behind threshold effects, corresponding to an asymmetric price transmission need to be developed and justified. Moreover, a great majority of empirical studies focus on the existence of asymmetry and, by and large, do not investigate the reason for its presence or absence (Bakucs and al., 2013). The theorical literature provides several explanations of why we should expect asymmetric price transmission. Asymmetric price transmission is the outcome of a market failure and these failures lead to a situation where the adjustment process of price transmission depends on a certain threshold. For example, policies such as price support mechanisms and tariff rate quotas may result in such adjustment process. In the former case, governments may intervene in the market when market prices fall below a threshold (floor price), while in the latter, international price signals are passed on when import volumes are sufficiently within, or out of the quota (Goshray, 2011).

A large number of studies conclude that market power, transportation costs, and government interventions are the main sources of threshold effects in price transmission. Other sources of threshold effects in price transmission include exchange rate, border policies, product homogeneity and differentiation, and inventories.

2.2.1 Market power

A commonly cited source of asymmetric price response is market power (Scherer and Ross, 1990). The market power of some agents may make them behave as price maker while others are price takers. Depending on their degree of oligopolistic influence, they adjust prices only when the adjustment allows to increase their commercial margins (Subervie, 2011). For instance, Oligopolistic middlemen in food markets might react quickly to price increases than to price reductions. However, if oligopolistic intermediaries are averse to price increases leading to a decrease in market share, they react more quickly to a price reduction than to a price increase (Cramon-Taubadel, 2004).

Moreover, the “so-called menu cost” thesis is pointed out by Blinder et al (1998). In fact, firms do not react to temporary price changes. Facing menu costs and inflation, firms may adjust their costs differently depending on whether prices are rising or falling (Ball and Mankiw, 1994). A cost increase leads to an extended gap between the real price and the desired price by quickly leading price adjustment, while costs reduction is always beneficial for firms.

The management of inventories is also a source of market failure, leading to threshold effects in price transmission. If stockholders anticipate a price increase in the central market, they increase their stocks in the hope of benefiting from the margins of a possible price increase. This leads to a supply increase on the local market at the time of the expected price increase. These two phenomena combined attenuate the transmission of the price increase. However, the same reasoning is relevant when stockholders anticipate a price reduction in central market. This leads to a supply decline in the local market at the time of the expected price decrease and to a moderate price decrease. Therefore, depending on the market power of oligopolistic middlemen, the way price increases and decreases in the central market affect the local market may differ and leads to asymmetric price transmission.

2.2.2 Government interventions

The immediate adverse effect of markets perturbation is the real income cut of the households who allocate the major part to buying agricultural food, particularly the poor. Indeed, the dramatic consequences of the food crises of the mid 70s and of 2007-2008 led many governments to adopt or strengthen specific trade policies to tackle the negative effects of price transmission. Trade policies affect price transmission through intervention instruments such **as tariffs, nontariff measures, tariff rate quotas, prohibitive tariffs,** etc. Ad valorem tariffs are expected to behave as fixed or proportional transactions costs (Conforti, 2004). Government interventions to support price stability is common in agriculture (e.g. floor prices). It implements some threshold prices to consider the welfare of economic agents (wholesaler, retailers, producers, and consumers) and tackle external effects. Depending on their position, the reaction of price increase is not the same as the price decrease.

Kinnucan & Forker (1987) show that such political interventions can lead to asymmetric price transmission if wholesalers or retailers believe that price decreases are temporary, and price increases are more persistent.

2.2.3 Transaction costs

Spatial asymmetric price transmission might arise if the costs of transportation vary with the direction of trade. This can depend on the quality of transports, telecommunications, and infrastructure. If two locations are separated by asymmetric transportation costs, then price transmission will only appear to be asymmetric if trade flows do indeed reverse from time to time and price movements originating in one or both locations are predominantly positive or negative. If price movements are distributed evenly at both locations, then both faster (down-stream) and slower (up-stream) transmission will be distributed evenly as well (Cramon-Taubadel, 2010).

3. Traditional approaches to detect threshold effects:

Tong (1978, 1983) and Tong and Lim (1980) are the first authors to propose threshold autoregressive models. The major features of this class of models are limit cycles, amplitudes dependent frequencies, and jump phenomena (Tsay, 1989).

3.1 Tsay (1989)

Due to the lack of a suitable modelling procedure and the inability to detect and estimate the threshold value, Tsay (1989) proposes a simple procedure for testing and detecting threshold effects. The modelling procedure developed by Tsay (1989) is defined as follow:

$$Y_t = \varphi_o^{(j)} + \varphi^{(j)} X_t + a_t^{(j)} \tag{1}$$

$$X_t = \left(Y_{t-1}, \dots, Y_{t-p}\right) \tag{2}$$

$Z_t = \mathrm{X}_{it}$ (i $= 1, \dots,$ p) a set of possible transition variables, where X_{it} corresponds to the ith column.

$$r_{j-1} \leq X_{it-d} \leq r_j \tag{3}$$

J=1, …, k

Where k is the number of regimes separated by k-1 nontrivial threshold r_j; p is the AR order (may differ from regime to regime, and also from one dependent variable to another); d is the threshold lag (called the delay parameter by Tong), $\left\{a_t^{(j)}\right\}$ is a martingale difference sequence $\left(E\left(a_t^{(j)}|F_{t-1}\right) = 0;\ sup_t E\left(|a_t^{(j)}|^{\delta}|F_{t-1}\right) < \infty\ for\ some\ \delta > 2;\ F_{t-1}\ is\ a\ \sigma\ field\ generated\ by\ a_{t-1}^{(j)}: \sigma_j{}^2 = var(a_t^{(j)})\right)$.

Referring to Tsay (1989), two features are special interest in various applications and are related to outliers and model changes in linear time series analysis.

Firstly, the TAR model becomes the homogenous linear AR model if only $\sigma_j{}^2 = var(a_t^{(j)})$ are different for different regimes.

Secondly, the TAR model reduces to a random-level shift model if only the constant terms $\varphi_o^{(j)}$ are different for different js.

The modelling procedure developed by Tsay (1989) is divided into four main steps. It is presented as follow:

- **Step 1**: Select the AR order p and the set of possible threshold lags S (collection of possible values of d).

- **Step 2**: Fit arranged autoregressions for a given p and every element d of S (set of possible transition variables), and perform the threshold nonlinearity test fi (p, d). If the nonlinearity of the process is detected, select the delay parameter d (or the transition variable).
- **Step 3**: For given p and d, locate the threshold values by using the scatterplots.
- **Step 4:** Refine the AR order and threshold values, if necessary, in each regime by using linear autoregression techniques.

3.1.1 Selection of the optimal order p and the set of possible thresholds

The selection of the optimal order p may be done by two ways. On one hand, the partial autocorrelation function may provide a reasonable value of p. On the other hand, the Akaike criteria of information (AIC) is also a used method but it misleads when the process is nonlinear. Besides, the AR order can also be refined at the end of the procedure.

3.1.2 Detection of threshold effects

Tsay (1989) used the portmanteau test of nonlinearity of Petruccelli and Davies (1986). He also built upon an arranged autoregression and predictive residuals.

3.1.3 Arranged autoregression and predictive residuals

An arranged autoregression is an autoregression based on the values of a particular regressor. To see this, consider the case k = 2. Let π_i be the time index of the r observations with the smallest transition variable Z_i. The modified model is as follow:

$$Y_{\pi_i} = \varphi_o^{(1)} + \varphi^{(1)} X_{\pi_i} + a_t^{(1)} \text{ for the r first observations of } Z_{\pi_i} \tag{4}$$

$$Y_{\pi_i} = \varphi_o^{(2)} + \varphi^{(2)} X_{\pi_i} + a_t^{(2)} \text{ else} \tag{5}$$

$$\text{Where } max(1, 1 + p - d) \leq \pi_i \leq n$$

Note that the modelling procedure of Tsay (1989) does not require knowing neither the precise value of the threshold, nor the number of observations in the first regime.

Referring to Tsay (1989), the motivation of his modelling procedure is that if the threshold is known, then consistent estimates of the parameters could easily be obtained by estimating each regime. Since the threshold value is unknown, however, one must proceed sequentially. If the r first observations with Z_{π_i} are so large, the least square estimates of $\widehat{\varphi^{(1)}}$ are consistent for $\varphi^{(1)}$. In addition, the predictive residuals are white noise asymptotically and orthogonal to the regressors $\{Y_{\pi_i} | i \leq s\}$. On the other hand, when i arrives at or exceeds s the predictive residual for the observation with time index π_{s+1} is biased because of the model change.

Considering the arranged regression, let $\hat{\beta}_r$ be the vector of least squares estimates based on the r cases, P_r the associated X'X inverse matrix, and x_{r+1} the vector of regressors of the next observation to enter the autoregression, namely $Y_{\pi_{r+1}}$. Based on the recursive least square estimation, estimates of the coefficients can be computed efficiently as follow:

$$\hat{\beta}_{r+1} = \hat{\beta}_r + K_{r+1}\left[Y_{r+1} - \widehat{x'}_{r+1}\hat{\beta}_r\right] \tag{6}$$

$$D_{r+1} = 1.0 + x'_{r+1}P_r x_{r+1} \tag{7}$$

$$K_{r+1} = \frac{P_r x_{r+1}}{D_{r+1}} \tag{8}$$

$$P_{r+1} = \left(I - P_r \frac{x_{r+1}x'_{r+1}}{D_{r+1}}\right) P_r \tag{9}$$

And the predictive and standardized predictive residuals as follow:

$$\hat{a}_{r+1} = Y_{r+1} - \widehat{x'}_{r+1}\hat{\beta}_{r+1} \tag{10}$$

$$\hat{e}_{r+1} = \frac{\hat{a}_{r+1}}{\sqrt{D_{r+1}}} \tag{11}$$

3.1.4 Nonlinearity test

The originality of the Tsay test (1989) comparing to Petrucelli and Davies (1986)[3] test is that the only information we need in advance about the threshold is the transition variable. And, the transition variable is supposed to be the threshold variable for which the linearity is most strongly rejected. The Tsay (1989) proposed test supposes that we have the optimal lag p, the threshold transition variable (d is supposed to be the lag of the transition variable, n-h-1 observations in arranged regression with h=$max(1, 1+p-d)$. We suppose that the recursive regression begins with b[4] observations with the smallest transition variable Z_i, and then followed by n-h-b-1 predictive residuals estimates. To do so, these models below are estimated:

$$\hat{e}_{\pi_i} = \varphi_0 + \varphi * X_{\pi_i} + \varepsilon_{\pi_i} \tag{12}$$

i=b+1…n-h-1

and the computed associate F statistic is defined as follow:

$$\boldsymbol{\hat{F}(p,d)} = \frac{\left(\sum_{i=b+1}^{n-h-1}\hat{e}_t^{\,2} - \sum_{i=b+1}^{n-h-1}\hat{\varepsilon}_t^{\,2}\right)/(p`+1)}{\left(\sum_{i=b+1}^{n-h-1}\hat{\varepsilon}_t^{\,2}\right)/(n-h-1-b)} \tag{13}$$

[3]This test is based on cumulative sums of standardized residuals from autoregressive fits to the data.
[4]Tsay (1989) suggest considering $b = \frac{n}{10} + p$ in the case of a TAR model.

The more the F statistic is near to zero, the more we fail to reject the null hypothesis (linearity). Under the null hypothesis of linearity, $\widehat{\boldsymbol{F}}(\boldsymbol{p}, \boldsymbol{d})$ follows approximately an F distribution with **p+1** and **n-h-1-b degrees of freedom**.

It is worth noting that the modelling procedure of Tsay (1989) requires identifying the threshold variable meaning the transition variable. Indeed, the threshold variable is not always the lag of the dependent variable. This might be defined in the model among a possible set of threshold variables. We then select the transition variable for which linearity is most strongly rejected, i.e. the one that maximizes the probability of the non-linearity test.

The recursive estimation of arranged autoregressions allows to estimate recursive residuals and to calculate a number of recursive statistics (coefficients and Student) used for locating thresholds.

Locating the thresholds can be done by graphical analysis of the representations of the different recursive statistics (predictive residuals, student statistics…) as a function of the ordered transition variable. The first threshold corresponds to the first break of the different recursive statistics.

However, if the first threshold detected (this corresponds to two regimes), it is possible to repeat the process by considering the observations with the transition variable above the threshold. The procedure is repeated until no break is detected on the different recursive statistics or the remaining observations are too low to ensure consistent estimates.

However, the model refinement relies on the minimization information criteria (AIC for instance) by doing a grid search in the surroundings of the initialization parameters. It is important to be careful of outliers because they may lead to unbiased predictive residuals. Outliers in the positives extremes values of the transition variable are less problematic. However, if the threshold is in the initialization observations, Tsay method is not able to detect it. It is the same if the threshold value is too small or too large. This leads to conclude that Tsay method is sensitive to outliers and to smaller or larger thresholds.

3.2 Balke and Fomby (1997)

Balke and Fomby (1997) point out that discrete adjustments may equally apply to policy interventions. For instance, exchange rate management and commodity price stabilization are often characterized by discrete interventions. Indeed, prices are supposed to fluctuate within a given band. When they get too far to the target bands, authorities intervene by using appropriate mechanisms.

Balke and Fomby (1997) relies on these potentials discontinuous adjustments to develop threshold cointegration models. Referring to Tsay (1989) method, Balke and Fomby (1997) take into account nonstationary processes to drop out spurious regressions. We show below the simple threshold cointegration model developed by Balke and Fomby (1997) and the non-linearity test considered.

3.2.1 Simple Threshold cointegration model

This method relies on the Engle and Granger (1987) approach which considers a bivariate system (y_t , x_t) defined as follow:

$y_t + \alpha x_t = z_t$ *where* $z_t = \rho^{(i)} z_{t-i} + \varepsilon_t$; ε_t $i.i.d$ *with mean zero;* z_t *the deviation from the equilibrium* *(14)*

$y_t + \beta x_t = B_t$ *where* $B_t = B_{t-i} + \eta_t$; η_t $i.i.d$ *with mean zero; Bt the common stochastic trend of yt and xt* *(15)*

The equation above represents the equilibrium relationship between y_t and x_t. Rather than Engle and Granger (1987) who consider z_t as a linear stationary process, Balke and Fomby (1997) highlight that z_t follows a threshold autoregression as below:

$\boldsymbol{\rho^{(i)} = 1\ if\ |z_{t-1}| \leq \theta}$, *where* $\boldsymbol{\theta}$ *is a critical threshold* *(16)*

$\boldsymbol{\rho^{(i)} = \rho, with\ |\rho| < 1\ if\ |z_{t-1}| \geq \theta}$ *(17)*

When $|z_{t-1}| > \theta$, z_t become a stationary autoregression that tends to revert back to a constant mean and also x_t and y_t tend to move towards some equilibrium. However, if the deviation is less than the critical threshold, there is no cointegration relation between x_t and y_t because of the presence of a unit root when $|z_{t-1}| \leq \theta$.

Rewriting the first system leads to the following:

$\Delta y_t = \lambda_1^{(i)} z_{t-1} + v_{1t}$ *(18)*

$\Delta x_t = \lambda_2^{(i)} z_{t-1} + v_{2t}$ *(19)*

Where

$\boldsymbol{\lambda_1^{(i)} = \frac{-(1-\rho^{(i)})\beta}{(\beta-\alpha)}; \lambda_2^{(i)} = \frac{-(1-\rho^{(i)})}{(\beta-\alpha)}};$ *(20)*

$\boldsymbol{v_{1t} = \left[\frac{\beta}{\beta-\alpha}\right]\varepsilon_t; v_{2t} = \left[\frac{1}{\beta-\alpha}\right](\eta_t - \varepsilon_t)};$ *(21)*

$\lambda_1^{(i)}$ *and* $\lambda_2^{(i)}$ (error correction parameters) capture the way $\mathbf{y_t}$ and $\mathbf{x_t}$ respond to deviations from equilibrium relation. $\lambda_1^{(i)}$ *and* $\lambda_2^{(i)}$ are nonzero only if the deviations from the equilibrium exceed the threshold.

3.2.2 Testing for cointegration and nonlinearity test

Testing for threshold cointegration requires to better identify the null hypothesis. Balke and Fomby (1997) point out that cointegration is a global characteristic while threshold regimes are local characteristics. Considering no cointegration and linearity as null hypothesis leads to three alternative hypotheses (no cointegration and nonlinearity, cointegration and linearity, and cointegration and nonlinearity). Balke and Fomby (1997) focus on this last alternative hypothesis (threshold cointegration). This consideration leads to two problems. First, there is a nonstandard inference problem5. Secondly, the class of stationary threshold models may be too large to permit testing parametrically the no cointegration/ linear against the general threshold cointegration alternative. The threshold cointegration test of Balke and Fomby (1997) essentially takes the Engle-granger single equation approach to cointegration by examining the equilibrium error.

As we can see, this method of Balke and Fomby is based on these two steps:

- **Step 1: Testing for cointegration**

If there is a linear cointegration relationship between y_t and x_t, the standard time series analyses for linear cointegration remain valid asymptotically for the threshold cointegration case. The reason is that the nonlinear behavior of the equilibrium error z_t does not affect the order of integration of y_t and x_t. Balke and Fomby perform the two-stage procedure. At first, the presence of unit roots is tested in y_t and x_t, as well as the determination of the cointegration vector. Secondly, the unit roots test is performed to test the stationarity of the error correction term. The growing literature of unit root tests allows to detect the presence of unit root (Augmented Dickey Fuller, Philips-Perron (1988), Bierens and Guo (1993) and Kwiatkowski et al, (1992)). It is shown that the powers of these tests are relatively low against the threshold alternative when we have small samples (Balke and Fomby, 1997).

- **Step 2: testing for nonlinearity**

The test of the threshold cointegration is based on the threshold behavior of the equilibrium error. Balke and Fomby (1997) look at the structural break in the arranged autoregression of the equilibrium error observations. For a given set of threshold values, the estimation of the threshold autoregression by least squares and the computation of the squared error allow to compute the Wald-Statistic based on the hypothesis of no structural change versus the alternative of a unit root in a band $[-\boldsymbol{\theta}, +\boldsymbol{\theta}]$ with $\boldsymbol{\theta}$ the threshold. All combinations of possible threshold values must be considered. The estimated threshold is the one that minimizes the sum of squared errors. The test statistic considered is the maximum Wald statistic (sup-Wald) over the set of possible threshold values. This method is far from unified. It seems to have low power compared to Hansen (1996) and to bootstrapping tests, likely the CUSUM and the Tsay test.

5 Presence of unit root in the null hypothesis and the nuisance parameter (threshold) under the alternative hypothesis

3.3 Enders-Granger (1998)

Enders and Granger (1998) paid attention to threshold and Momentum threshold autoregressive processes. They describe a class of models that can be used as the basis of unit root tests in the presence of asymmetric adjustment. The contribution of Enders and Granger (1998) is the introduction of momentum threshold autoregressive, commonly called the M-TAR, models. Indeed, this allows a variable to display differing amounts of autoregressive decay depending on whether it is increasing or decreasing.

3.3.1 TAR and M-TAR model

Enders and Granger (1998) defined the higher-order processes presented as follows to test the presence of asymmetric adjustment. Let $\{z_t\}$ a process and

$$\Delta \boldsymbol{z_t} = \boldsymbol{a_0} + \boldsymbol{I_t \rho_1 z_{t-1}} + (\boldsymbol{1} - \boldsymbol{I_t})\boldsymbol{\rho_2 z_{t-1}} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{p-1}} \boldsymbol{\beta_i} \Delta \boldsymbol{z_{t-1}} + \boldsymbol{\varepsilon_t}\ (\boldsymbol{E}) \tag{22}$$

Where p is supposed to be the appropriate lag length; I_t is the Heaviside indicator function such that:

$I_t = \begin{cases} 1\ if\ z_{t-1} \geq 0 \\ 0\ if\ z_{t-1} \leq 0 \end{cases}$ in a case of a TAR model or $I_t = \begin{cases} 1\ if\ \Delta z_{t-1} \geq 0 \\ 0\ if\ \Delta z_{t-1} \leq 0 \end{cases}$ in a case of a MTAR model.

Considering the TAR specification, the adjustment depends on the level of z_{t-1}. About the MTAR specification, the adjustment depends on the previous period level change in z_{t-1}. Besides, the MTAR exhibits more momentum in one direction than other. Indeed, increases tend to persist but decreases tend to revert quickly toward the attractor.

3.3.2 Testing for unit root versus TAR and MTAR adjustments

Enders and Granger (1998) perform the following four steps to test the stationary of the series and suggest a model building procedure to estimate the threshold.

- **Step 1: choice of the specification**

As in the Dickey-Fuller test, the unit root test proposed by Enders and Granger requires a special testing procedure. Indeed, there is no simple way to jointly test the presence of the deterministic regressors (intercept or trend) and the unit root. When regression residuals are used; the appropriate attractor is $z_t=0$. However, when the presence of deterministic regressors (intercept or trend) is in doubt, an ad hoc procedure is suggested to fit the model.

- **Step 2: Estimation of the model and computation of the F statistic**

There is no straightforward way to determine a unified estimation method. The type of attractor and the type of asymmetry (TAR or MTAR) under consideration are determinant when it comes to proposing the way to estimate ρ_1 and ρ_2 as defined in (E) and to computing the F statistic for the null hypothesis $\rho_1=\rho_2$. The comparison between this statistic test with the appropriate critical values allows to determine if the null hypothesis of a unit root test can be rejected.

- **Step 3: Checking the residuals**

The residuals are supposed to be a white noise process. If the residuals are correlated, it might be better getting back to step 3 and estimate the model with a better appropriate p lag. There are several ways to determine the lag lengths. Model-selection criteria such as the Akaike information criterion (AIC) or Bayesian information criterion (BIC) are the most used.

- **Step 4: Chan (1993) or Tsay (1989) to fit the best asymmetric adjustment**

Enders and Granger (1998) suggest using Chan (1993) or Tsay (1989) approaches to fit the best asymmetric adjustment model. Enders and Granger (1998) used the MTAR and found that the Chan (1993) method gives a consistent estimator of the threshold. Comparing this to other methods (TAR, pre-specified attractor), the consistent estimator of the threshold fits substantially better.

To sum up, Enders and Granger (1998) look into the presence of a unit root in case of asymmetric adjustment and propose an alternative way to handle this fact. It is shown that unit root tests have low power in the presence of asymmetric price transmission. Besides, they introduced the MTAR model to capture the way the magnitude of positive and negative previous changes of the long-term equilibrium error leads to different adjustments.

However, Enders and Siklos (2001) point out the possibility of a multivariate analysis with the TAR and the MTAR models and propose a generalized threshold autoregressive (TAR) and momentum-TAR tests for unit-roots in a multivariate context.

4. New approaches to detecting threshold effects

4.1 Enders-Siklos (2001)

Balke and Fomby (1997) and Enders and Granger (1998) show that unit-roots and cointegration tests all have low power in the presence of asymmetric adjustment. Besides, Engle-Granger (1987) and Johansen (1996) tests are based only on a linear adjustment. Then if this hypothesis falls, the relevancy of using these methods breaks down. However, Enders and Siklos (2001) propose an extension of the Engle-Granger testing strategy by permitting asymmetry in the adjustment toward equilibrium in two different ways. They show that the test has good power and size properties over the Engle-Granger test when there are asymmetric departures from equilibrium.

4.1.1 Enders and Siklos (2001) approach

Based on a linear and an asymmetric adjustment, the considered relationship is as follow:

$$\Delta x_t = \pi x_{t-1} + v_t \tag{23}$$

$$\Delta x_{it} = \alpha_i (x_{1t-1} - \beta_o - \beta_{2t-1} x_{2t-1} - \cdots - \beta_n x_{nt-1}) + v_{it} \text{ (error correction representation} \tag{24}$$

$$x_{1t1} = \beta_o + \beta_{2t} x_{2t-1} + \cdots + \beta_n x_{nt-1} + \mu_{t-1} \text{ (long-run equilibrium relationship)} \tag{25}$$

Where:

x_t is an (nx1) vector of random variables all integrated of degree 1,

π is an (nxn) matrix,

v_t is an (nx1) vector of the normally distributed disturbances v_{it} that may be contemporaneously correlated.

Considering this relationship valid, Engle and Granger (1987) and Johansen (1988) propose a test of cointegration. Engle and Granger (1987) adopt the two-step procedure (2OLS) while Johansen (1988) is based on the estimation of the π matrix and the rank test (trace test) or the maximum eigenvalues test (Johansen and Juselius (1990)) to test cointegration.

Enders and Siklos (2001) point out that these cointegration tests and their extensions are mis-specified if adjustment is asymmetric.

When the adjustment is asymmetric, in the case of Engle and Granger (1987 approach, the second step which consists of testing the stationarity of μ_t as follows $\Delta\mu_t = \rho\mu_{t-1} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{p-1}} \boldsymbol{\beta_i} \Delta\mu_{t-1} + \boldsymbol{\varepsilon_t}$ is mis-specified.

Enders and Siklos (2001) consider the alternative specification of the error-correction model, called the threshold autoregressive (TAR) model, defined as follows:

$$\Delta\mu_t = \boldsymbol{\rho_1 I_t \mu_{t-1}} + \boldsymbol{\rho_2 (1 - I_t) \mu_{t-1}} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{p-1}} \boldsymbol{\beta_i} \Delta\mu_{t-1} \tag{26}$$

Where: $I_t = \begin{cases} 1 \ if \ \mu_{t-1} \geq 0 \\ 0 \ if \ \mu_{t-1} < 0 \end{cases}$

An alternative adjustment specification is the momentum threshold autoregressive (MTAR) model, defined below:

$$\Delta\mu_t = \rho_1 I_t \mu_{t-1} + \rho_2 (1 - I_t) \mu_{t-1} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{p-1}} \boldsymbol{\beta_i} \Delta\mu_{t-1} + \boldsymbol{\varepsilon_t} \tag{27}$$

$$I_t = \begin{cases} 1 \ if \ \Delta\mu_{t-1} \geq 0 \\ 0 \ if \ \Delta\mu_{t-1} < 0 \end{cases}$$

In case of an asymmetric adjustment, the error correction model for any variable x_{it} can be written in the form below:

$$\Delta \boldsymbol{x_{it}} = \boldsymbol{\rho_{1i} I_t \mu_{t-1}} + \boldsymbol{\rho_{2i}(1 - I_t)\mu_{t-1}} + \cdots + \boldsymbol{v_{it}} \quad (28)$$

ρ_{1i} and ρ_{2i} are the adjustment coefficients for positive and negative discrepancies.

4.1.2 Cointegration test with TAR and MTAR adjustment

To perform the Enders and Siklos (2001) cointegration test, the four steps modelling procedure presented below allows to perform the Engle and Granger (1987) cointegration test in case of asymmetric adjustment.

- **Step 1: Cointegration relationship estimation**

This consists in regressing one of the variables on a constant and the covariates and save the long-run equilibrium error $\widehat{\mu_t}$. Depending on the type of asymmetric adjustment (TAR or MTAR), it is possible to estimate ρ_{1i} and ρ_{2i} and compute the F-statistic under the null hypothesis $\rho_{1i} = \rho_{2i} = 0$. Then, the alternative hypothesis is accepted, when the computed F-statistic is up to the critical values computed by Enders and Siklos (2001).

- **Step 2: Adjustment testing**

When we accept that $\widehat{\mu_t}$ is a stationary under the alternative hypothesis, it is possible to test symmetric against asymmetric adjustment. This test is based on this null hypothesis ($\rho_{1i} = \rho_{2i}$) corresponding to the symmetric adjustment. The F-statistic can be used to test the null hypothesis.

- **Step 3: Diagnostic of the residuals**

The residuals $\widehat{\varepsilon_t}$ are supposed to be characterized by a white-noise process. In case of serial correlation, one should get back to step 1 and re-estimate the model by adjusting the lag length p using appropriate selection criteria (AIC or BIC).

- **Step 4: Performing estimations**

The attractors (threshold) are supposed be equal to zero a priori. Yet Tong (1983) shows that this assumption can be a biased estimate of the attractor. In doing so, it is possible to use Chan (1993) or Tsay (1989) to detect endogenously the threshold.

Enders and Siklos (2001) noted that:

- All the tests for the MTAR model have at least as much power than those for the corresponding TAR model.
- If adjustment is truly symmetric, Engle and Granger test is more powerful than alternative tests considering asymmetric effect.

- The higher the degree of asymmetry, the more powerful the tests considering asymmetric adjustment.

4.2 Hansen-Seo (2002)

Hansen and Seo (2002) noted that the existing threshold cointegration tests separate the estimated cointegrating vector and the prespecified threshold. This means that both computations are really separated. The contributions of Hansen and Seo (2002) are twofold. First, they propose a maximum likelihood estimation (MLE) procedure but don't proof the consistency of the estimates. Their method is based on a grid search over the threshold and the cointegrating vector. Second, they develop a test for the presence of a threshold effect. In doing so, the conventionnel VECM (linear VECM) is supposed to be the null hypothesis. Under the null hypothesis, the Lagrange multiplier (LM) principle is applied to compute the statistic test. Based on the grid search, they consider the Sup_{LM} test.

4.2.1 MLE Estimation approach

In case of a linear cointegration, the VECM model can be represented as follows:

$$\Delta x_t = A' X_{t-1}(\beta) + u_t \quad (29)$$

Where:

$X_{t-1}(\beta) = (1, w_{t-1}(\beta), \Delta x_{t-1}, \Delta x_{t-2}, \dots, \Delta x_{t-l})'$;

x_t, a p-dimensional I (1) time series cointegrated (β the cointegration vector);

$w_{t-1}(\beta) = \beta' x_t$, the long-run error correction term.

A a (kxp) matrix with k=pl+2.

u_t is supposed to be i.i.d gaussian with finite covariance matrix $\sum = E(u_t u'_t)$.

The parameters $(\beta, A, \sum)$ are estimated by maximum likelihood under the assumption supposed above.

In case of threshold cointegration, a two-regime threshold vector error correction model can be represented as follow:

$$\Delta x_t = A'_1 X_{t-1}(\beta) I_t + (1 - I_t) A'_2 X_{t-1}(\beta) + u_t \quad (30)$$

$$I_t = \begin{cases} 1 \ if \ w_{t-1}(\beta) \leq \delta \\ 0 \ if \ w_{t-1}(\beta) > \delta \end{cases}$$

Where δ is the threshold parameter.

The threshold effect exists whether $0 < P(w_{t-1}(\beta) \leq \delta) < 1$, otherwise the model simplifies to linear cointegration.

Hansen and Seo (2002) impose a trimming parameter π_0 such as $\pi_0 < P(w_{t-1}(\beta) \leq \delta) < 1 - \pi_0$ and propose the maximum likelihood method under the assumption made on the error u_t (gaussian, and i.i.d). The gaussian likelihood is as follow:

$$L_n(A_1, A_2, \sum, \beta, \delta) = -\frac{n}{2} log(|\Sigma|) - \frac{1}{2}\sum_{t=1}^{n} u_t(A_1, A_2, \beta, \delta)'\Sigma^{-1} u_t(A_1, A_2, \beta, \delta) \qquad (31)$$

Where

$$u_t(A_1, A_2, \beta, \delta) = \Delta x_t - A'_1 X_{t-1}(\beta) I_t - (1 - I_t) A'_2 X_{t-1}(\beta) \qquad (32)$$

In this approach, $\beta \; and \; \delta$ are unknown. Hansen and Seo (2002) perform a grid search by estimating a linear model and suggesting an interval where $\hat{\beta}$ fluctuates and another interval where δ is supposed to be in. These choices are supposed to verify the following condition: $\pi_0 < \frac{1}{n}(\sum_{t=1}^{n} 1(w_{t-1}(\beta) \leq \delta)) < 1 - \pi_0$.

For each value of $\beta \; and \; \delta$ in these intervals, the optimal parameters are those who minimize the $log(|\Sigma|)$.. Hansen and Seo (2002) noted that when $\hat{\beta}$ and $\widehat{\delta}$ converge to the real values of the parameters, the slope estimates $\widehat{A_1} \; and \; \widehat{A_2}$ should have conventional normal asymptotic distributions.

4.2.2 Testing for a threshold

The testing method proposed by Hansen and Seo (2002) relies on the null hypothesis of linear VECM model and the alternative hypothesis of one threshold VECM model and also consider a number of regimes larger than two. They consider the LM statistics since it is easy to compute and to bootstrap, and a likelihood or Wald test would require a distribution theory for the parameter estimates of the unrestricted model.

In theory, under the null hypothesis:

$$\Delta x_t = A' X_{t-1}(\beta) + u_t \qquad (33)$$

Under the alternative hypothesis:

$$\Delta x_t = A'_1 X_{t-1}(\beta) I_t + (1 - I_t) A'_2 X_{t-1}(\beta) + u_t \qquad (34)$$

$$I_t = \begin{cases} 1 \; if \; w_{t-1}(\beta) \leq \delta \\ 0 \; if \; w_{t-1}(\beta) > \delta \end{cases}$$

Unlike the Loglikelihood and the Wald tests, which are used to assess the change in model fit when more than one variable is added to the model, the Lagrange multiplier has the particularity to test the expected change in model fit if one or more parameters which are currently constrained are allowed to be estimated freely.

In this way, Hansen and Seo (2002) use the following likelihood function under the threshold effect.

$$L_n(\beta,\delta) = L_n\left(\widehat{A_1},\widehat{A_2},\sum,\beta,\delta\right) = -\frac{n}{2}log(|\Sigma|) - n \tag{35}$$

The threshold value is supposed to be unknown under the null hypothesis and the β parameter is either estimated or known a priori or supposed to be in an interval. Regardless of the approach about the prespecified cointegration vector, the LM statistic must be defined as follows:

$$SupLM = Sup_{\delta_L \leq \delta \leq \delta_U} LM(\beta,\delta) \tag{36}$$

Hansen and Seo (2002) point out that the δ parameter estimate with the maximum likelihood estimation method is not the same with the δ parameter estimate by the Supremum Lagrange Multiplier, which is robust to heteroskedasticity.

However, the asymptotic distribution of the LM test is unknown. It is approximated by means of a bootstrapping simulation method. Hansen and Seo (2002) perform the Hansen (1996) approach using fixed regressors bootstrap, which is not expected to provide a better approximation to the finite sample distribution than conventional asymptotic approximations. The advantage of this method is that it allows for heteroscedasticity of unknown form and provides heteroscedasticity-consistent standard errors. Indeed, Hansen and Seo (2002) highlight that it is difficult to incorporate heteroscedasticity in a conventional bootstrap, namely residuals bootstrap (using i.i.d innovations). Usual bootstrap will fail to achieve the first-order asymptotic distribution unlike the fixed regressors bootstrap, which does.

4.3 Non-linear ARDL (Shin et al (2011))

Shin et al (2011) highlight a famous Keynes (1936, p.314) remark, which noted that "the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when an upward is substituted for a downward tendency". They noted that, more generally, the assumption of linear adjustment may be excessively restrictive in a wide range of economically interesting situations, particularly where transaction costs are non-negligible and where policy interventions are observed in-sample.

However, they noted that most papers modelling short-run asymmetry employ the two step Engle-Granger technique which is inherently less efficient than single-step ECM estimation, and the scarcity of papers modeling long and short-run asymmetric jointly too. By the way, *Manera and Frey (2005) pointed out that: "in general, a SR analysis is more indicated to compare the intensity of output price variations to positive or negative changes in input prices, whereas a LR perspective is needed if the empirical investigation concentrates on the computation of reaction times, length of fluctuations, as well as speeds of adjustment toward an equilibrium level".*

In that framework, Shin et al (2011) developed a simple and flexible nonlinear dynamic framework capable of simultaneously and coherently modelling asymmetries both in the underlying long-run relationship and in the patterns of dynamic adjustment. They presented three main steps to set up the nonlinear ARLD modelling procedure.

- Step 1: Define the Nonlinear ARDL approach
- Step2: Set up the bounds-testing procedure
- Step 3: Trace out the asymmetric adjustment patterns

4.3.1 Nonlinear ARDL cointegration approach

The Shin et al. (2011) nonlinear ARDL cointegration approach is an asymmetric approach of the well-known ARDL model of Pesaran and Shin (1999) and Pesaran et al. (2001) to capture both long- and short-run asymmetries in a variable of interest and require either I(0) or I(1) variables, or a mixture of both. However no I(2) variable should be present. Considering y_t as the variable of interest, x_t as a system of kx1 vector of explanatory variables defined such that $x_t = x_0 + x_t^+ + x_t^-$, where $x_t^+ = \sum_{j=1}^{t} max(\Delta x_j, 0)$; $x_t^- = \sum_{j=1}^{t} min(\Delta x_j, 0)$; the nonlinear ARDL model is defined as follow:

$$\boldsymbol{y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \sum_{j=0}^{q} \left(\theta_j^{+\prime} x_{t-j}^+ + \theta_j^{-\prime} x_{t-j}^-\right) + \varepsilon_t} \tag{37}$$

Where $\theta_j^+ \; and \; \theta_j^-$ are the asymmetric distributed-lag parameters and ε_t a white noise time series.

Referring to Pesaran and Shin (1999) and Pesaran et al. (2001), Shin et al. (2011) consider the error correction model as follow:

$$\begin{aligned} \Delta y_t &= \rho y_{t-1} + \theta^{+\prime} x_{t-1}^+ + \theta^{-\prime} x_{t-1}^- + \sum_{j=1}^{p-1} \lambda_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\phi_j^{+\prime} \Delta x_{t-j}^+ + \phi_j^{-\prime} \Delta x_{t-j}^-\right) + \varepsilon_t \\ &= \rho \xi_{t-1} + \sum_{j=1}^{p-1} \lambda_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\phi_j^{+\prime} \Delta x_{t-j}^+ + \phi_j^{-\prime} \Delta x_{t-j}^-\right) + \varepsilon_t \end{aligned} \tag{38}$$

Where $\rho = \sum_{j=1}^{p-1} \phi_j - 1$; $\lambda_j = -\sum_{i=j+1}^{p-1} \phi_i$ for j=1,…,p-1; $\theta^+ = \sum_{j=1}^{q} \theta_j^+$; $\theta^- = \sum_{j=1}^{q} \theta_j^-$;

$\phi_0^+ = \theta_0^+$; $\phi_j^+ = -\sum_{i=j+1}^{q-1} \theta_j^+$ for j=1,…,q-1; $\phi_j^- = -\sum_{i=j+1}^{q-1} \theta_j^-$ for j=1,…,q-1, and

$\xi_{t-1} = y_t - \beta^{+\prime} x_{t-1}^+ - \beta^{-\prime} x_{t-1}^-$ the nonlinear error correction term where $\beta^+ = -\theta^+/\rho$; $\beta^- = -\theta^-/\rho$ are the associated asymmetric long-run parameters.

4.3.2 Bounds-testing Asymmetric cointegration

The bounds-testing procedure proposed by Shin et al (2001) can be performed using two approaches. First, referring to Banerjee, Dolado and Mestre (1998), the null hypothesis of no long-run relationship is $\rho = 0$

against $\rho < 0$. Second, based on Pesaran, Shin and Smith (2001), the null hypothesis of no long-run relationship becomes $\rho = \theta^{+} = \theta^{-} = 0$. The test statistics are denoted t_{BDM} and F_{PSS}, respectively.

Depending to the way (t_{BDM} or F_{PSS} statistics) one performs the test of the no long-run relationship, Pesaran, Shin and Smith (2001) tabulated two critical value bounds depending on the number of regressors entering in the long-run relationship. The lower bound assumes that all regressors are I (0), while the upper bound supposes that they are all I (1). If the computed statistic above the upper bound, the no cointegration hypothesis is rejected. If the computed statistic is down to the lower bound, cointegration is rejected. However, if computed statistic falls within the lower and the upper bounds, further investigation is needed.

4.3.3 Asymmetric cumulative dynamic multiplier effects

When the presence of asymmetric cointegration is not rejected, it is important to look into the change in x_t^{+} and x_t^{-} on y_t. The cumulative dynamic multiplier effects of x_t^{+} and x_t^{-} are derived as follows:

$$m_h^{+} = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial x_t^{+}};\ m_h^{-} = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial x_t^{-}};\ h=0,1,2\ldots \tag{39}$$

$$With\ \lim_{h\to\infty} m_h^{+} = \beta^{+} = -\theta^{+}/\rho;\ \lim_{h\to\infty} m_h^{-} = \beta^{-} = -\theta^{-}/\rho$$

Where $\beta^{+} = -\theta^{+}/\rho$ and $\beta^{-} = -\theta^{-}/\rho$ are the asymmetric long-run coefficients.

Unlike the other approaches of asymmetric cointegration relationships analyzed so far, the NARDL model presents interesting features. Indeed, as noted by Shin et al (2011), NARDL models admit three general form of asymmetry:

- long-run or reaction asymmetry, associated with $\beta^{+} \neq \beta^{-}$;
- impact asymmetry associated with the inequality of the coefficients on the contemporaneous first differences Δx_t^{+} and Δx_t^{-} (i.e, $\sum_{j=0}^{q-1} \phi_j^{+} \neq \sum_{j=0}^{q-1} \phi_j^{-}$;
- adjustment asymmetry captured by the patterns of adjustment from initial equilibrium to the new equilibrium following an economic perturbation (i.e. the dynamic multipliers).

4.4 A model-based recursive partitioning approach

One of the main problems of traditional methods for detecting thresholds are their inabilities to capture several asymmetric adjustments. Machine learning methods overcome these shortcomings by allowing to detect several thresholds with less computational power.

As highlighted by Zeileis, Hothorn and Hornik (2008), the models based on regression trees analysis have two main features:

- Interpretability: enhanced by visualizations of the fitted decision trees and
- Predictive power in non-linear regression relationships.

Model-based recursive partitioning is a special case and an extension of regression trees that allows to test for non-linearities in regression. The approach proposed by Zeileis, Hothorn and Hornik (2008) is particularly interesting as it i) can detect more than one threshold and ii) encompasses many instability and structural change tests such as the Lagrange multiplier tests or the score tests (Zeileis, 2005).

The model-based recursive partitioning approach can be described by three main steps as follows: 1) parameter estimation; 2) testing parameter instability; 3) splitting

4.4.1 Parameter estimation

For a general (parametric) model $M(Y,\theta)$ with observations Y, and a k-dimensional vector of parameters θ, the standard model can be fitted by minimizing the objective function below:

$$\hat{\theta} = Argmin_{\theta \in \Theta}\left(C(y,\theta) = \sum_{i=1}^{n} \psi(Y_i,\theta)\right) \tag{40}$$

Various estimation techniques can be used like the ordinary least squares (OLS) or maximum likelihood (ML) among M-type estimators to estimate $\hat{\boldsymbol{\theta}}$.

The recursive partitioning algorithm consist of determining the optimal partition $\{B_{b^*}\}^6$ by doing a greedy forward search where the objective function ψ can at least be optimized locally in each step. However, if we have only one splitting variable, the optimal split can be found easily.

4.4.2 Testing for parameter instability

As we can see from the first step, the basic idea being that each node is associated with a single model, the estimation is first performed for all observations through the minimization of the cost function $C(Y,\theta)$.

At the current node, parameter instability is tested with respect to the splitting variables $Z_1, \dots, Z_l$. The instability is assessed via a Generalized M-fluctuation test that encompasses most of the structural change tests found in the literature (Zeileis, 2005)[7]. When instability cannot be rejected, the Z_j associated with the highest parameter instability is selected and the sample split with respect to that particular Z_j so that $C(Y,\theta)$ is locally optimized.

To assess the parameter instability, a natural idea is to check whether the scores $\hat{\varphi}_l = \frac{\partial \boldsymbol{\psi}(\mathbf{Y}, \hat{\boldsymbol{\theta}}_l)}{\partial \theta_l}$ fluctuate randomly around their mean 0 or exhibit systematic deviations from 0 over Z_j.

Theses deviations can be captured by:

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[6] $Z = Z_1 * Z_2 * \dots * Z_l$ with $\{Z_l\}_{l=1,2,\dots}$ the partition variables (also namely the splitting variables)

[7] Including OLS CUSUM and MOSUM tests (Chu, Hornik, and Kuan, 1995), score tests (Hjort and Koning 2002) and Lagrange multiplier statistics (Andrews and Ploberger 1994).

$$W_j(t) = \hat{J}^{-\frac{1}{2}} * n^{-\frac{1}{2}} * \sum_{i=1}^{[nt]} \hat{\varphi}_{\sigma(Z_{ij})} \, (0 \le t \le 1) \tag{41}$$

$$\hat{J} = n^{-1} * \sum_{i=1}^{n} \psi(Y_i, \hat{\theta}) \psi(Y_i, \hat{\theta})^{\intercal} \tag{42}$$

This idea above allows to capture the instabilities over a numerical splitting variable $Z_j$by using the supremum of LM statistics (Sup_{LM}) against a single change point alternative defined below:

$$\lambda_{Sup_{LM}(W_j)} = \max_{i=i_{min}\cdots i_{max}} \left(\frac{i}{n} . \frac{n-i}{n}\right)^{-1} \left\| W_j\left(\frac{i}{n}\right) \right\|_2^2 \tag{43}$$

The limiting distribution of the Sup_{LM} test statistic is given by the supremum of a Bessel process from which p-values are computed for each particular ordering of Z_j. To avoid small sample size, in each node a certain fraction (10%) of the current observations are trimmed on each end.

The approach developed by Zeileis, Hothorn and Hornik (2008) combines both ideas (from (linear) model tree algorithms, such as GUIDE (Loh 2002) or the RD and RA trees of Potts and Sammut (2005)).

4.4.3 Splitting

The process is repeated for the remaining variables to generate daughter nodes and is stopped if no significant instabilities is found. However, the numbers of splits can either be fixed or determined adaptively. Indeed, performing an exhaustive search over all conceivable partitions with B segments is guaranteed to find the optimal partition but might be burdensome. In doing so, several search methods exist for numerical and categorical splitting variables. In case of a categorical variable, the number of splits cannot be larger than the number of categories.

5. Application: the rice market in Senegal

5.1 Data, unit root, cointegration and Seasonality tests

5.1.1 Data

In this technical note, we focus on the Dakar ordinary broken rice price and the Thailand A1 rice price to perform the price transmission analysis described above. The price transmission analysis is performed at retail level in Dakar, the most important segment of the Senegalese market. In Senegal, the ordinary broken rice segment with 100% breakage is consumed by about 80% of broken rice consumers. Monthly prices paid by final consumers from 1995 to 2014[8] are given by the Commission for Food Security. Regarding the

[8] Price series present discontinuities after 2014 for the local market in Dakar.

Thai A1 rice price, it comes from the World Bank Commodity Price Data and expressed in US dollars. This corresponds to the world reference FOB price. The reference market considered is the Bangkok market.

As part of the price analysis, the price series are taken in logarithm so that coefficients represent elasticities. It is worth noting that, when we mention domestic price, we refer to the Dakar ordinary broken rice price, and world price refers to Thailand A1 rice price.

5.1.2 Testing for unit roots and cointegration

To perform cointegration tests, it is important to test firstly the presence of unit roots. The Augmented Dickey-Fuller (ADF) and the Phillips-Perron tests are used for it and as a robustness check, the KPSS test for stationarity is also performed.

Since we will be working with monthly data, we also used seasonal unit root tests to detect a stochastic seasonal behavior which cannot be handled by dummy variables. Hylleberg, Engle, Granger and Yoo (HEGY) (1990) proposed a procedure to detect unit roots at zero and seasonal frequencies and also to determine the appropriate differentiation filter to make the new series stationary. HEGY's (1990) test statistics developed on quarterly series have been extended to monthly data by Franses (1991) and Beaulieu and Miron (1993). These tests were completed by Ghysels et al. (1994); they allow to test the presence of unit roots at all seasonal frequencies simultaneously with or without the zero frequency. (Darné et al., 2002).

Table 1: Unit root tests' results

		Log (Rice Thai A1's price)		**Log (Broken rice Dkr price)**	
		Test statitistic	***p-value***	**Test statitistic**	***p-value***
Augmented Dickey-Fuller test	**Level**	-1.87	0.38	-2.41	0.16
	First difference	-11.63	0.01	-14.23	0.01
Phillips-Perron test	**Level**	-12.2	0.35	-15.4	0.21
	First difference	-162	0.01	-236	0.01
KPSS test	**Level**	0.83	0.01	0.41	0.01
	First difference	0.04	0.1	0.04	0.1

Source: Authors' calculation

Both the Augmented Dickey-Fuller and the Phillips-Perron tests cannot reject the null hypothesis of the presence of a unit root for series in level. When applied to the first-differenced series, both tests strongly reject the null hypothesis (p-value=0.01<0.05). Therefore, we conclude that both series contain a unit root. These results are confirmed by the KPSS test which reverse the null hypothesis. For series in level, we reject the stationarity assumption and cannot reject it for first differences.

The HEGY test for seasonal unit roots rejects the null hypothesis for all frequencies except 0 (Table 2). The zero frequency corresponds to the annual one, therefore we confirm again the results of the Dickey-Fuller, Phillips-Perron and KPSS tests.

Table 2: Seasonal unit root (HEGY) tests[9]

		Log(Broken rice Dakar)	**Log(Rice Thaï A1)**
Frequencies	**Coefficients**	**Student's statistic**	
0	$\pi 1$	-1.429	-1.179
π	$\pi 2$	-5.588***	-4.199 ***
		Fisher statistics	
π/2	π3 = π4	20.952***	32.563***
2π/3	π5 = π6	16.283 ***	15.271***
π/3	π7 = π8	13.943 ***	28.166***
5π/6	π9 = π10	17.219 ***	26.394***
π/6	π11 = π12	35.403 ***	15.743***

Note: *** means p-value is less than 0.01
Source: Authors' calculation

The previous steps have established that both the world (Thailand) and domestic (Senegal) price of rice are integrated of order one, and only at the annual frequency. We can therefore test for cointegration. We perform both residual based approaches (Engle-Granger and Phillips Ouliaris) and the bounds test of Pesaran Shin and Smith (2001).

Table 3 presents the results based on the residuals-based tests of cointegration[10]. The null hypothesis of both tests is that the residuals contain a unit root. The null hypothesis is rejected for both tests even at the 1% level. This result is confirmed by the bounds test since the F-statistic is above the upper I (1) bound (see Table 4). We therefore cannot reject cointegration between the domestic and world prices.

Table 3: Residuals based cointegration tests

	Test statistic	**Critical values for 1%**		
Engle-Granger	-4.23	1pct	5pct	10pct
Philipps-Ouliaris	-4.17	-3.46	-2.87	-2.57

Source: Authors' calculation

[9] The test is performed through the General Dickey-Fuller regression of the form:

$$(1 - L^{12})y_t = \alpha + \sum_{i=1}^{p} \varphi_i \Delta_{12} y_{t-i} + \sum_{k=1}^{12} \pi_k y_{k,t-1} + \varepsilon_t$$

Given that for all the frequencies except 0 and π, the corresponding roots come in conjugate pairs, a joint test must be implemented for each pair.

[10] The residuals are from the long run equation (first step of the Granger approach to test cointegration). The long run elasticity of the domestic price with respect to world price is 0.43, adopting the Granger approach, and is relatively smaller compared to ARDL approach (0.50).

Table 4: ARDL bounds tests

Test statistic (F)	1% critical value I (0) bound	1% critical value I (1) bound
8.56	5.15	6.36

Source: Authors' calculation

5.2 Nonlinearity tests

5.2.1 Hansen-Seo (2002)

To detect the threshold effect in price transmission between domestic and world price of rice, we first perform the Hansen-Seo (2002) approach. This approach developed below relies on an a priori known cointegrating vector (as tested above). This leads to a sequential testing procedure, consisting of the estimation of TVECMs based on a prespecified number of regimes (here 2 regimes) and the grid-search over the threshold parameters. Figure 1 shows that the estimated threshold is 0.05[11]. Based on the Fixed regressor bootstrap and the standard residual bootstrap experiments, the p-values for the SupLM statistic (19.9) are 0.018 and 0.034, respectively. This result means an asymmetric price transmission between domestic and world price of rice.

[11] The threshold variable is the residuals of the long run relation with trend and intercept

Figure 1: Sum of squared residuals (SSR) with potential thresholds using grid search

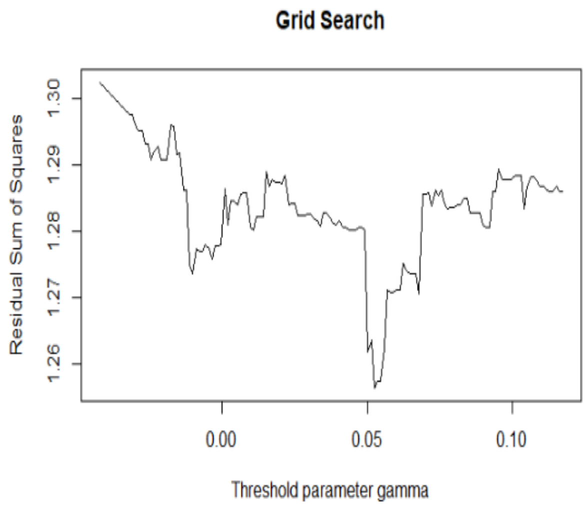


Figure 2: Test of linear versus threshold cointegration

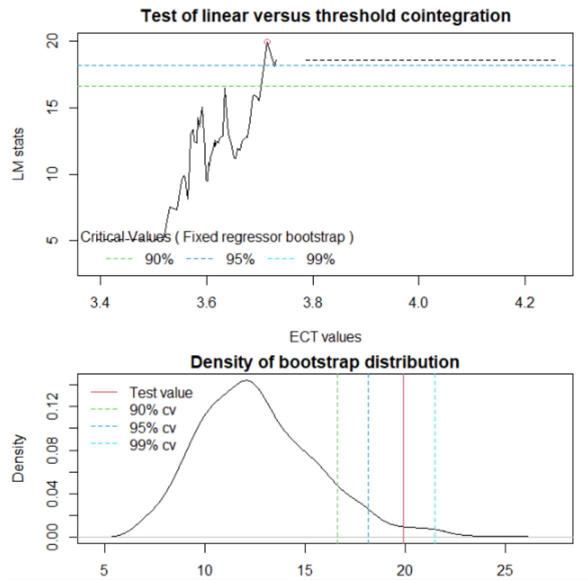


*Table 5**:** Test of linear versus threshold cointegration of Hansen and Seo (2002)*

Cointegration vector (1, -0.38) Threshold estimate= 0.05 LM statistic test (SupLM)= 19.9	P-value (Fixed regressor bootstrap) = 0.018 P-value (Residuals bootstraps) = 0.034

Source: Authors' calculation

Table 6 present the threshold vector error correction model (TVECM) with a cointegrating parameter equals to 0.4. This implies that a 10 percent increase in the world price of rice results in 4 percent increase in the domestic price of rice. The estimated cointegrating parameter is obtained by an OLS estimation procedure with intercept and trend.

Regarding the relation between the domestic and the world price, the results in table 6 show that the adjustment of the domestic price is faster when disequilibria are negative and slower when disequilibria are positive. Indeed, 12.5% of positive deviations (i.e, a drop in the world price of rice corresponding to a positive value of the long-term margin or an increase up to 0.05), are absorbed within one month while 23,5% of negative deviations (i.e, an increase in the international price of rice corresponding to the negative value of the long-term margin) are eliminated after one month. The TVECM results indicate also that there is no reverse effect when domestic price of rice changes in accordance with the small country assumption. This fact supports the evidence of vertical price transmission between world and domestic price of rice.

Table 6: Threshold Vector error correction model estimation

Dependent variable	**dLog_Price_SEN**		**dLog_Price_THAI**	
Coefficients	Estimates (+)	Estimates (-)	Estimates (+)	Estimates (-)
ECT	-0.1252***	-0.2355**	-0.0668	0.2691*
	(0.0002)	(0.0078)	(0.1496)	(0.0296)
Const	0.4573***	0.8508**	0.2506	-0.9644*
	(0.0003)	(0.0073)	(0.1466)	(0.0295)
dLog_Price_SEN_1	0.0607	0.2078	0.0059	-0.1636
	(0.4173)	(0.0748)	(0.9551)	(0.3150)
dLog_Price_THAI_1	-0.2565***	0.0304	0.0683	0.4096***
	(0.0005)	(0.6179)	(0.4994)	(2.9e-06)

Source: Authors' calculation

5.2.2 Non-linear ARDL (Shin et al (2011))

As seen above, TVECM models do not provide information about the long-run asymmetries and focus on the way the adjustment takes place in the short-run. When this assumption of linear long-run relation fails, the results about the adjustment in short-run are also problematic. Shin et al (2011) developed the nonlinear ARDL estimation procedure allowing to analyze long and short-run dynamic relations. As we can see in Table 7 below, the results show evidence of nonlinear cointegration. Indeed, the computed statistic is higher than the upper bound (F_PSS=15.78>4.10). Then, the no cointegration hypothesis is rejected. We also observe in Table 7 a symmetry in the impact of the world price of rice on the domestic price (positive and negative changes) in the long-run (p-value=0.966), but an asymmetric adjustment exists in the short-run (p-value=0.000).

Table 7: Bounds-test for Nonlinear Cointegration, and asymmetric test (short-run and long-run)

Bounds-test for Nonlinear Cointegration			
Test statistic	Value	1% critical value I (0) bound	1% critical value I (1) bound
F_PSS	15.78	3.43	4.1

Asymmetric test			
Long-run asymmetry $H0: (\theta^{+} = 0.066) = (\theta^{-} = 0.066)$		Short-run asymmetry $H0: \left(\sum_{j=0}^{q-1} \phi_j^{+} = 0.632\right) = \left(\sum_{j=0}^{q-1} \phi_j^{-} = 0.724\right)$	
F-stat	P>F	F-stat	P>F
.002	0.966	40.07	0.000

Source: Authors' calculation

Table 8: Long-run effect

Dependent variable	**Log_Price_SEN**					
Effects	Long-run effect [+]($\beta^{+} = -\theta^{+}/\rho$)			Long-run effect [-]($\beta^{-} = -\theta^{-}/\rho$)		
	coef.	F-stat	P>F	coef.	F-stat	P>F
Log_Price_THAI	0.36	30.88	0.000	0.36	22.86	0.000

Source: Authors' calculation

The results of the estimation in Table 9 suggest the NARDL model successfully captures the short-run asymmetry in the responses of the domestic price of rice to changes in the world price. The domestic price of rice is more sensible to lagged negative changes than to lagged positive changes. This can be seen when looking at in the values of the short-run coefficients presented in Table 7 (0.63 versus 0.72).

Table 9: Asymmetric Error correction model

Dependent variable	**dLog_Price_SEN**
	Estimates
P^{D}_{t-1}	-0.182***
	(0.027)
$P^{m}_{t-1}(+)$	0.066***
	(0.016)
$P^{m}_{t-1}(-)$	0.066***
	(0.017)
ΔP^{D}_{t-1}	0.030
	(0.057)
ΔP^{D}_{t-2}	0.018
	(0.058)
ΔP^{D}_{t-3}	0.131*
	(0.057)
$\Delta P^{m}_{t}(+)$	0.202**
	(0.066)
$\Delta P^{m}_{t-1}(+)$	0.004
	(0.069)
$\Delta P^{m}_{t-2}(+)$	0.013
	(0.070)
$\Delta P^{m}_{t-3}(+)$	0.415***
	(0.070)
$\Delta P^{D}_{t}(-)$	0.067
	(0.094)
$\Delta P^{D}_{t-1}(-)$	-0.322***
	(0.096)
$\Delta P^{D}_{t-2}(-)$	-0.153
	(0.098)
$\Delta P^{D}_{t-3}(-)$	-0.316**
	(0.098)
Cons	0.968***
	(0.145)
N	229
R2	0.308
F	8.254
Model diagnostics	Stat (p-value)
Portmanteau test up to lag 40 (chi2)	52.7(0.09)

Notes: Signif. codes: '***' means p-value < 0.001; '**' p- value < 0.01; '*' means p- value < 0.05
Source: Authors' calculation

5.2.3 A model-based recursive partitioning approach vs Enders and Siklos approach

We note that the tools developed above to detect asymmetric effects appear to fix either a well-known threshold or the number of thresholds. The model-based recursive partitioning approach gives the opportunity to detect endogenously the thresholds and does not suppose a priori the number of thresholds. In doing so, we perform the model-based partitioning approach of Zeiles, Hothorn and Hornik (2008) to the residuals of the cointegration relationships to detect potential thresholds (asymmetries) in the relationship between the domestic and world prices. Furthermore, we compare this latter approach with the Enders and Siklos approach (2001) to detect threshold effects (asymmetry). In both procedures we use the same autoregressive specification to detect endogenously the threshold. The general model is given by: $\Delta\nu_t = I_t\, \rho_1 \nu_{t-1} + (1 - I_t)\rho_2 \nu_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta\nu_{t-i} + u_t$

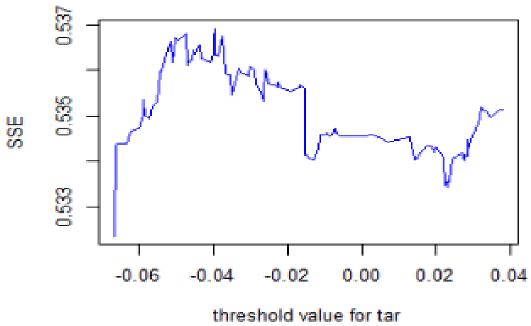
However, while the Enders and Siklos (2001) approach determines the threshold by minimizing the sum of the squares of the residuals performing either the Chan (1993) or the Tsay (1989) approach, the model-based partitioning approach of Zeiles, Hothorn and Hornik (2008) relies on the parameters instability test given by the supremum of LM statistics (Sup_{LM}).

The results presented in Figure 3 and Figure 4 show the presence of a threshold effect (asymmetry) in both approaches. The negative threshold value shows that a new adjustment takes place after a substantial reduction of the margin. The two thresholds computed differently by performing an Enders and Siklos (2001) and the model-based recursive partitioning approaches are very close and are respectively -0.068 and -0.053. These results are consistent with previous findings in the price transmission analysis which suggest that symmetric price transmission is the exception, but not the rule (Peltzman, 2000).

It is worth noting that the Enders and Siklos (2001) approach performs the asymmetric test by testing whether the coefficients of adjustment $\rho_1\ et\ \rho_2$ are significantly different and negative. Table 10 supports this fact. Regarding the model-based recursive partitioning, the procedure of testing instability of parameters, presented in Table 11 shows that there is one threshold, and that it is not possible to performing a new split.

Enders and Siklos approach

Figure 3: Sum of squared residuals (SSR) with potential thresholds



Model-based recursive partitioning

Figure 4: Recursive partitioning tree

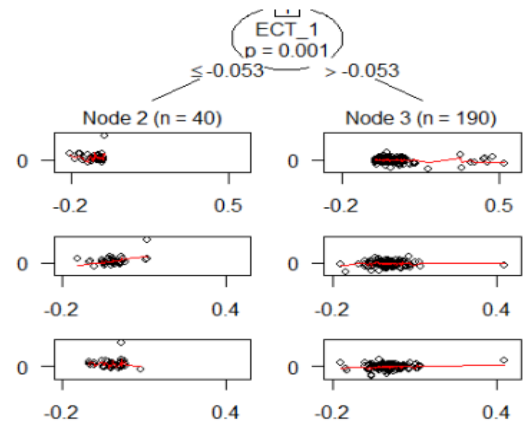


Table 10: Enders and Siklos Asymmetric test

H0	$\rho_1 = \rho_2$	$\rho_1 = \rho_2 = 0$
F-statistic	4.2443	11.233
P-value	0.04052 *	2.225e-05 ***

Table 11 : Fluctuation tests of splitting variables

Fluctuation tests of splitting variables			
Nodes	Threshold	statistic	p.value
Node 1	-0.053	20.255	0.0011
Performing split		3.28	0.84

Note: Note: ECT_1 refers to the lagged residuals of the long-run relationship, i.e ν_{t-1}
Note: Signif. codes: '***' means p-value < 0.001; '**' p- value < 0.01; '*' means p- value < 0.05
Source: Authors' calculation

Based on the results above, we observe that the speed of adjustment differs from price increases to price decreases. Therefore, we estimate the following asymmetric error correction model with threshold cointegration.

$$\Delta P_t^D = I_t\,\lambda_1 \nu_{t-1} + (1 - I_t)\lambda_2 \nu_{t-1} + \sum_{i=1}^{p} \alpha_1(i)\Delta P_{t-1}^D + \sum_{i=0}^{p} \alpha_2(i)\Delta P_{t-1}^m + \epsilon_t \qquad (44)$$

Where P_t^D is the logarithm of the Dakar ordinary broken rice price and P_t^m, the logarithm of the Thailand A1 rice price.

The results in Table 12 show that domestic price reacts more rapidly when the deviation[12] from the equilibrium is squeezed than when it is stretched. Referring to the Enders and Siklos (2001) approach, only 12.50% of positive deviations (international prices go down or go up with a margin not lower than -0.06)

[12] The error correction term

are eliminated at the end of the subsequent month, while 33.2% of negative deviations (world prices go up) are eliminated after one month. As to the model-based partitioning, these figures are 16.1 and 32.8% respectively with a threshold value of -0.053 for world price increases.

Table 12: Asymmetric error correction model estimation

Dependent variable: $\Delta \boldsymbol{P_t^D}$		
Approaches	**Enders and Siklos**	**Model-based recursive partitioning**
Coefficients	**Estimates**	**Estimates**
Const (-)	-0.108	-0.175
	(0.122)	(0.116)
Const (+)	-0.09	-0.18
	(0.121)	(0.118)
ΔP_{t-1}^D	0.016	0.033
	(0.022)	(0.021)
ΔP_t^m	0.121**	0.109**
	(0.048)	(0.046)
ΔP_{t-1}^m	-0.109**	-0.144***
	(0.049)	(0.048)
ECT_1 (-)	-0.332**	-0.328**
	(0.156)	(0.162)
ECT_1 (+)	-0.125***	-0.161***
	(0.039)	(0.044)
Observations	231	231
Threshold	-0.068	-0.053
R2	0.125	0.175
Adjusted R2	0.097	0.149
Residual Std. Error (df =	0.044	0.043
F Statistic (df = 7; 224)	4.554***	6.777***

Notes: Signif. codes: '***' means p-value < 0.001; '**' p- value < 0.01; '*' means p- value < 0.05
Source: Authors' calculation

As we can see in Table 12 above, the model-based recursive partitioning approach yields better results compared to Enders and Siklos (2001) in terms of model diagnostics. Even if the difference is not large, but it is important to set up both approaches in order to check the robustness of the results.

It is worth noting that these methods of detection of threshold effects are sensitives to outliers and structural breaks. Consequently, it is a good practice to look into the series of prices and consider the potential presence of outliers and structural breaks.

6. Conclusion

The analysis of price transmission plays a key role in understanding markets integration. This leads to better identifying the nature of relationship between geographically distant markets and cross-commodity price transmission, analyzing the impact of liberalization policies, as well as the identification of regions exposed to systemic shocks.

This technical note focuses on the tools available to researchers to analyze asymmetric price transmission and threshold effects. Asymmetric price transmission relies on the fact that price adjustment differs from price increases to price decreases. This contributes to the debate between symmetric and asymmetric adjustment by presenting the methods detecting asymmetry effects and showing the advantages and limitations of the main tools used by researchers in the area.

It is worth noting that, there is no one-size-fits-all method to detect asymmetric effects. The analyst needs to select a combination of elements (context of study, the economy under consideration, data availability…) to justify the relevancy of their choice. However, over the two decades, the flagship methods used in the literature to detect threshold effects in price transmission are the new approaches[13] developed after 2000. These new approaches, except the model-based recursive partitioning (2008), are the extensions of the traditional approaches, like Tsay (1989), Balke and Fomby (1997), and Enders-Granger (1998).

One of the main differences of the approaches developed in this note is the nature of the asymmetry (long-run versus short-run) and the way the threshold effects are detected. Enders and Siklos (2001) test the asymmetry in the short-run and identify endogenously the threshold by performing a Tsay (1989) approach or a Chan (1993) approach. The Hansen-Seo (2002) method provides a multidimensional framework and determines endogenously the threshold by using either a known cointegration vector or not. By applying a machine learning algorithm to the traditional asymmetric cointegration model in the Enders-Siklos (2001) approach, the model-based recursive approach of Zeileis, Hothorn and Hornik (2008) allows to detect endogenously the thresholds and does not require the number of pre-existing thresholds. Regarding the nonlinear ARDL (2011) model, it allows the user to test both the long-run and the short-run asymmetries but supposes a priori an existing threshold.

Finally, we note that the results from all methods performed in the case of the rice market in Senegal show the evidence of an asymmetric price transmission between world and domestic prices in the short-run and a symmetric transmission in the long-run. However, the researcher should check if the price series contain outliers or present structural breaks. It is also worth noting that the methods did not analyze why these asymmetries occur. The researcher must mobilize the literature. The main raisons of asymmetry are market power, transaction costs, and government interventions.

[13] Enders and Siklos (2001), Hansen-Seo (2002), Model-based recursive partitioning (2008), Non-linear ARDL (2011)

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